INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 5.9]

RELATED RATES

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 8]

The quantity A increases at a constant rate $\frac{dA}{dt} = 3$.

(a) Given that $C = 2A^3 + 1$, find the rate of change of C, at the instant when $A = 2$; [3]

(b) Given that $\ln D = \frac{3}{A}$, find the rate of change of D, at the instant when $D = e$; [5]

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2. [Maximum mark: 5]

The quantities A and B increase at constant rates $\frac{dA}{dt} = 3$ and $\frac{dB}{dt} = 2$ respectively.

Given that $F = 2\text{A}^2 B + 2B^3$, find the rate of change of F , when $A = B = 1$.

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3. [Maximum mark: 6]

The quantities A and B increase at constant rates $\frac{dA}{dt} = 3$ and $\frac{dB}{dt} = 2$ respectively.

Given that $F^4 = 2\text{A}^2 B + 2B^3$, find the rate of change of F , when $A = B = 1$.

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4. [Maximum mark: 6]

Air is pumped into a spherical ball which expands at a rate of 8 cm^3 per second (8 cm^3s^{-1}).

Find the **exact** rate of increase of the radius of the ball when the radius is 2 cm.

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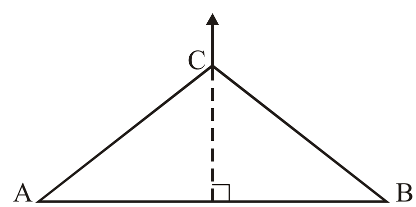
5. [Maximum mark: 6]

The following diagram shows an isosceles triangle ABC with AB = 10 cm and AC = BC. The vertex C is moving in a direction perpendicular to (AB) with speed 2 cm per second.



Calculate the rate of increase of the angle CAB at the moment the triangle is equilateral.

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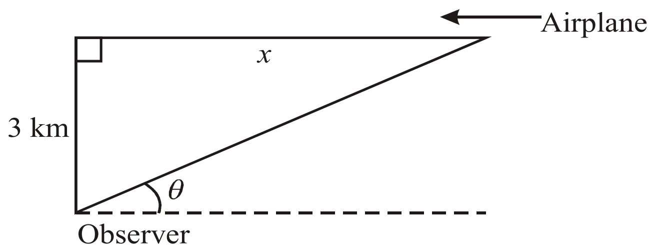
6. [Maximum mark: 6]

An airplane is flying at a constant speed at a constant altitude of 3 km in a straight line that will take it directly over an observer at ground level. At a given instant the observer notes that the angle θ is $\frac{1}{3}\pi$ radians and is increasing at $\frac{1}{60}$ radians per second.



Find the speed, in kilometres per hour, at which the airplane is moving towards the observer.

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7. [Maximum mark: 5]

In the previous problem, find the rate of change of the distance between the observer and the airplane, at the instant when the angle θ is $\frac{1}{3}\pi$ radians and is increasing at $\frac{1}{60}$ radians per second.

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8. [Maximum mark: 6]

A conical tank with vertex down is 8 metres in diamater and 12 meters deep. Water flows into the tank at 10 m^3 per minute. Find the rate of change of the depth of the water at the instant when the water is 6 meters deep.

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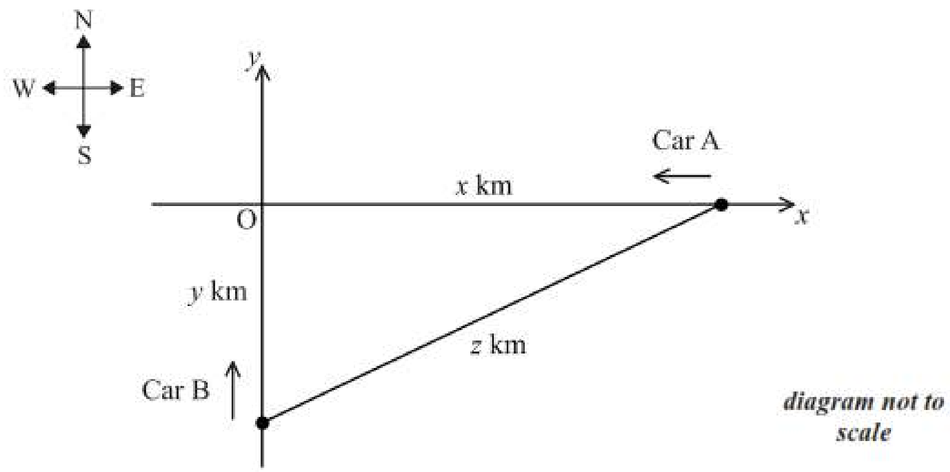
9. [Maximum mark: 6]

Car A is travelling on a straight east-west road in a westernly direction at 60 km h^{-1}.

Car B is travelling on a straight north-soutg road in a northernly direction at 70 km h^{-1}.

The roads intersect at the point O. When Car A is x km east of O, and Car B is y km south of O, the distance between the cars is z km.



Find the rate of change of z when Car A is 0.8 km east of O and Car B is 0.6 km south of O.

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10. [Maximum mark: 5]

In problem 9, answer the same question if Car A was travelling in an easternly direction.

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11. [Maximum mark: 6]

The volume of a solid is given by $V = \frac{4}{3}\pi r^3 + \pi r^2 h$.

At the time when the radius is 3 cm, the volume is 81π cm^3, the radius is changing at a rate of 2 cm/min and the volume is changing at a rate of 204π cm^3/min. Find the rate of change of the height at this time.

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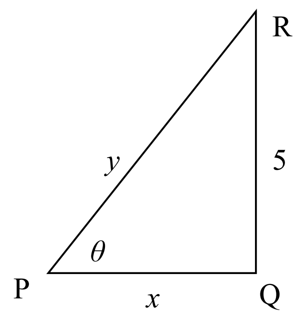
B. Paper 2 questions (LONG)

12. [Maximum mark: 24]

The diagram shows a right-angled triangle PQR, with $\hat{Q} = \frac{\pi}{2}$ and QR = 5 m (constant).

The sides PQ = x and PR = y are variable, as P can be moved horizontally on the line (PQ).

Hence the angle $\theta = R\hat{P}Q$, the area A and the perimeter P of the triangle are also variable.



(a) Complete the following table

Variables	Relation between the **variables**	Relation between the corresponding **rates of change**
x and θ	$\tan\theta = \frac{5}{x}$	$\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = -\frac{5}{x^2}\frac{dx}{dt}$
y and θ		
x and y		
A and x		
P, x and y		

[12]

(b) At the instant when $x = 5$ m, write down the values of

(i) θ (ii) y (iii) A (iv) P [4]

(c) Given that P is moving to the left by 0.5 m per second, find the rate of change of the following at the instant when $x = 5$ m

(i) x (ii) y (iii) A (iv) P [8]

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13. [Maximum mark: 12]

The height and the diameter of the base of a cylinder are equal.

(a) Express the volume V of the cylinder in terms of the radius r. [1]

(b) Show that the surface area S of the cylinder is given by $S = 6\pi r^2$. [2]

(c) Find the relations between

(i) the rates $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}r}{\mathrm{d}t}$. (ii) the rates $\frac{\mathrm{d}S}{\mathrm{d}t}$ and $\frac{\mathrm{d}r}{\mathrm{d}t}$. [4]

(d) **Hence**, find the relation between the rates $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}S}{\mathrm{d}t}$. [2]

The volume of the cylinder increases at a constant rate of 6 $\text{cm}^3\ \text{min}^{-1}$.

(e) Find the rate of change of the surface area at the instant when r = 12 cm. [3]

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