

INTERNATIONAL BACCALAUREATE  
Mathematics: applications and interpretation

**MAI**

**EXERCISES [MAI 5.4]**  
**MONOTONY – MAX – MIN**  
Compiled by Christos Nikolaidis

**A. Paper 1 questions (SHORT)**

1. [Maximum mark: 12]

Differentiate the following functions and hence determine whether each function is increasing or decreasing (or neither).

(i)  $f(x) = x^3 + x + 5$

(ii)  $f(x) = 5 - x - x^3$

(iii)  $f(x) = 2x^5 + \sqrt{x}, x > 0$

(iv)  $f(x) = \frac{x^2}{2} - 2x, x > 2$

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2. [Maximum mark: 8]

Consider the function  $f(x) = x^3 + 3x^2 - 9x$  which passes through the origin.

- (a) Find  $f'(x)$ . [2]
- (b) **Hence** find the  $x$ -coordinates of any stationary points and determine their nature. [4]
- (c) Sketch the graph of the function. Indicate the stationary points. [2]

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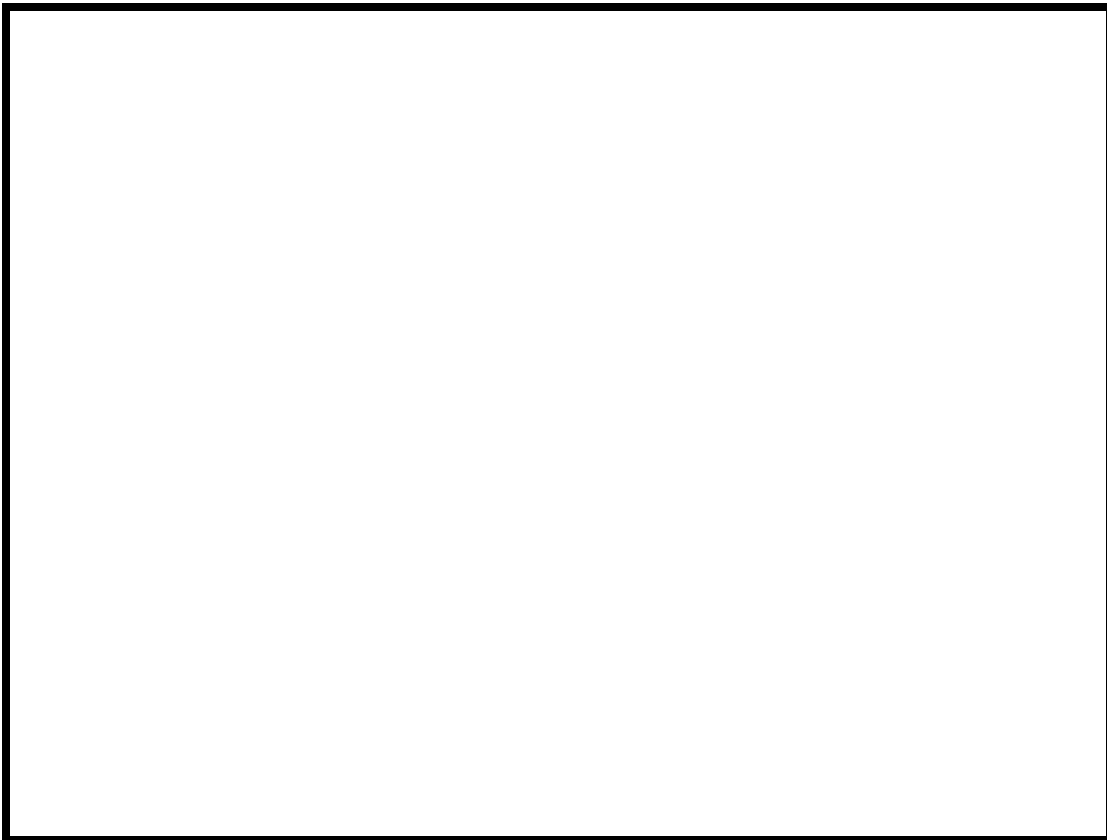
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3. [Maximum mark: 8]

Consider the function  $f(x) = x^3 + 3x^2 + 3x$

- (a) Find  $f'(x)$ . [2]
- (b) **Hence** find the  $x$ -coordinates of any stationary points and determine their nature. [4]
- (c) Sketch the graph of the function. Indicate the stationary points. [2]

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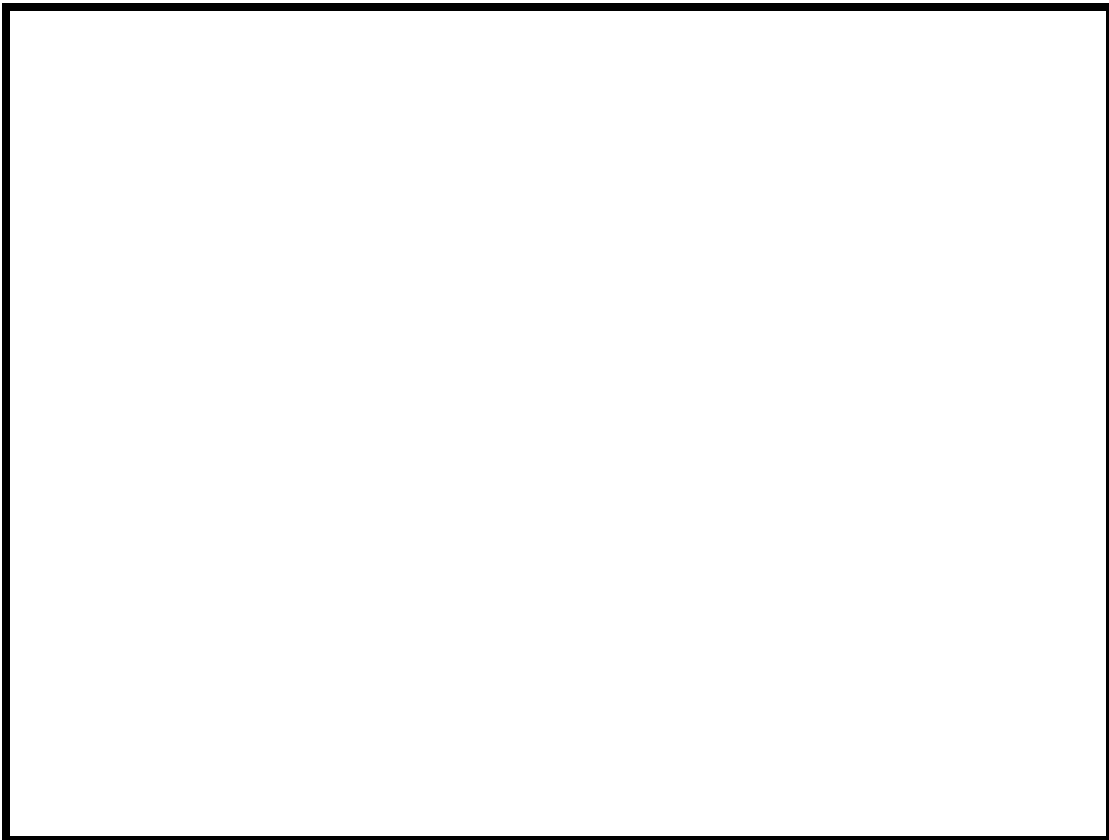
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4. [Maximum mark: 8]

The function  $f(x)$  is given by  $f(x) = x^3 - 3x^2 + 3x$ , for  $-1 \leq x \leq 3$ .

(a) Differentiate  $f(x)$  with respect to  $x$ . [2]

(b) Copy and complete the table below.

$x$	-1	0	1	2	3
$f(x)$		0	1	2	9
$f'(x)$	12		0		12

[3]

(c) Use the information in your table to sketch the graph of  $f(x)$ . [2]

(d) Write down the gradient of the tangent to the curve at the point (3, 9). [1]

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5. [Maximum mark: 7]

The function  $g$  is defined as follows

$$g : x \mapsto px^2 + qx + c, \quad p, q, c \in \mathbb{R}$$

- (a) Find  $g'(x)$  [2]
- (b) If  $g'(x) = 2x + 6$ , find the values of  $p$  and  $q$ . [2]
- (c)  $g(x)$  has a minimum value of  $-12$  at the point A. Find
  - (i) the  $x$ -coordinate of A;      (ii) the value of  $c$ . [3]

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6. [Maximum mark: 8]

A function  $g(x) = x^3 + 6x^2 + 12x + 18$ .

- (a) Find  $g'(x)$ . [2]
- (b) Solve  $g'(x) = 0$ . [2]
- (c) (i) Calculate the values of  $g'(x)$  when (a)  $x = -3$ ; (b)  $x = 0$ .  
(ii) Hence state whether the function is increasing or decreasing at  
(a)  $x = -3$ ; (b)  $x = 0$ . [4]

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7. [Maximum mark: 5]

It is given that  $f'(x) = (x-1)(x-3)$ .

Find the stationary points of  $f$  and determine their nature.

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8. [Maximum mark: 5]

It is given that  $f'(x) = (x-1)(x-3)(x-5)^2$ .

Find the stationary points of  $f$  and determine their nature.

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9. [Maximum mark: 7]

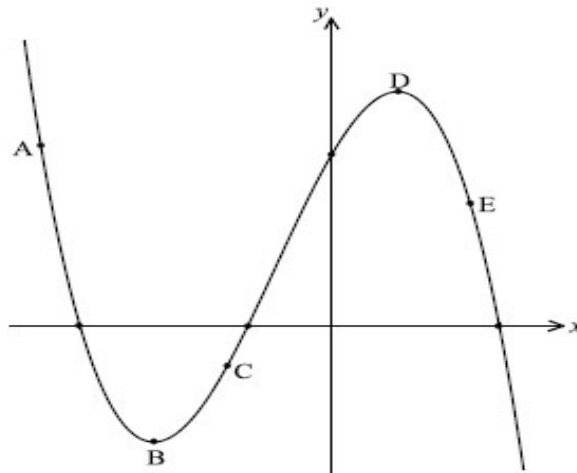
Let  $g(x) = x^3 - 3x^2 - 9x + 5$ .

- (a) Find the two values of  $x$  at which the tangent to the graph of  $g$  is horizontal. [4]  
 (b) For each of these values, determine whether it is a maximum or a minimum. [3]

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10. [Maximum mark: 6]

The following diagram shows part of the curve of a function  $f$ . The points A, B, C, D and E lie on the curve, where B is a minimum point and D is a maximum point.



- (a) Complete the following table, noting whether  $f'(x)$  is positive, negative or 0 at the given points.

	A	B	C	E
$f'(x)$				

[4]

- (b) Complete the following table: note whether each value is positive, negative or 0.

$f(0)$	$f'(0)$

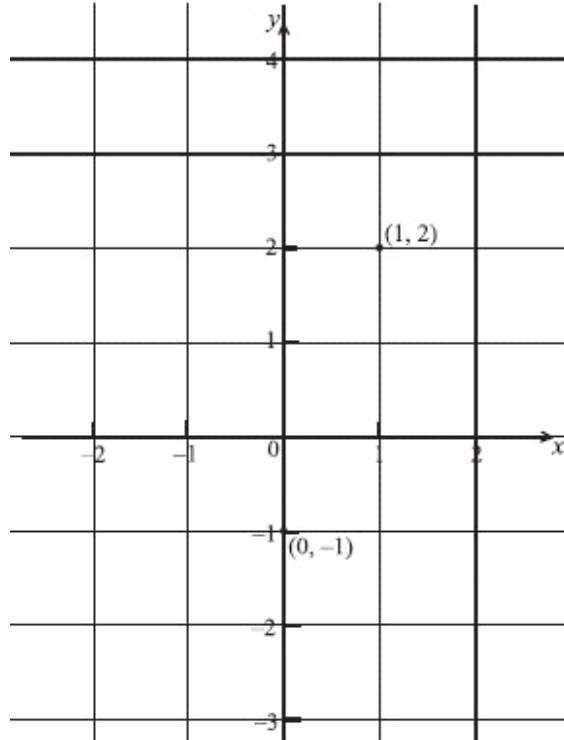
[2]

11. [Maximum mark: 7]

For each case below, sketch on the axes a curve  $y = f(x)$  which satisfies the conditions of the table on the left.

(a)

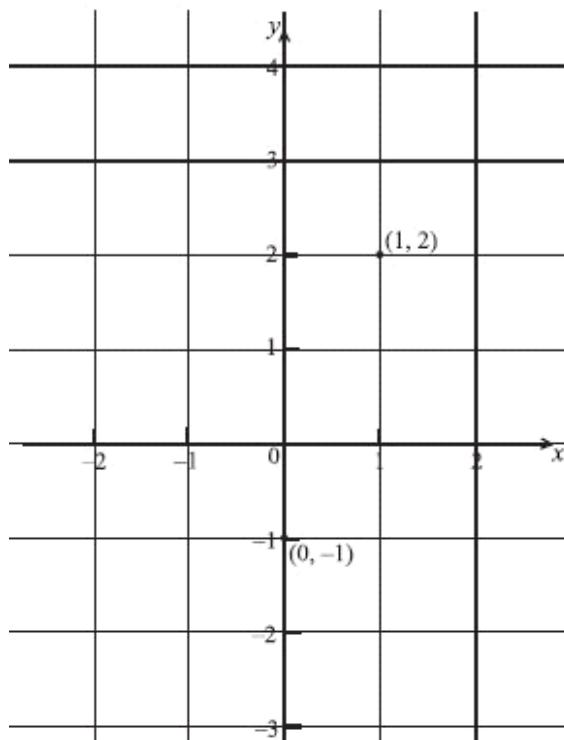
$x$	$f(x)$	$f'(x)$
-2	2	
$-2 \leq x < 0$		negative
0	-1	0
$0 < x < 1$		positive
1	2	0
$1 < x \leq 2$		negative



[3]

(b)

$x$	$f(x)$	$f'(x)$
-2	2	
$-2 \leq x < 0$		negative
0	-1	0
$0 < x < 1$		positive
1	2	0
$1 < x \leq 2$		positive



[4]



12. [Maximum mark: 6]

The function  $g(x)$  is defined for  $-3 < x < 3$ . The behaviour of  $g'(x)$  is given in the table below.

$x$	$-3 < x < -2$	$-2$	$-2 < x < 1$	$1$	$1 < x < 3$
$g'(x)$	negative	0	positive	0	negative

Use the information above to answer the following. In each case, justify your answer.

- (a) Write down the value of  $x$  for which  $g$  has a maximum. [1]
- (b) On which intervals is the value of  $g$  decreasing? [2]
- (c) Given that  $g(-3) = 1$ , sketch the graph of  $g$ . On the sketch, clearly indicate the position of the maximum point and the minimum point. [3]

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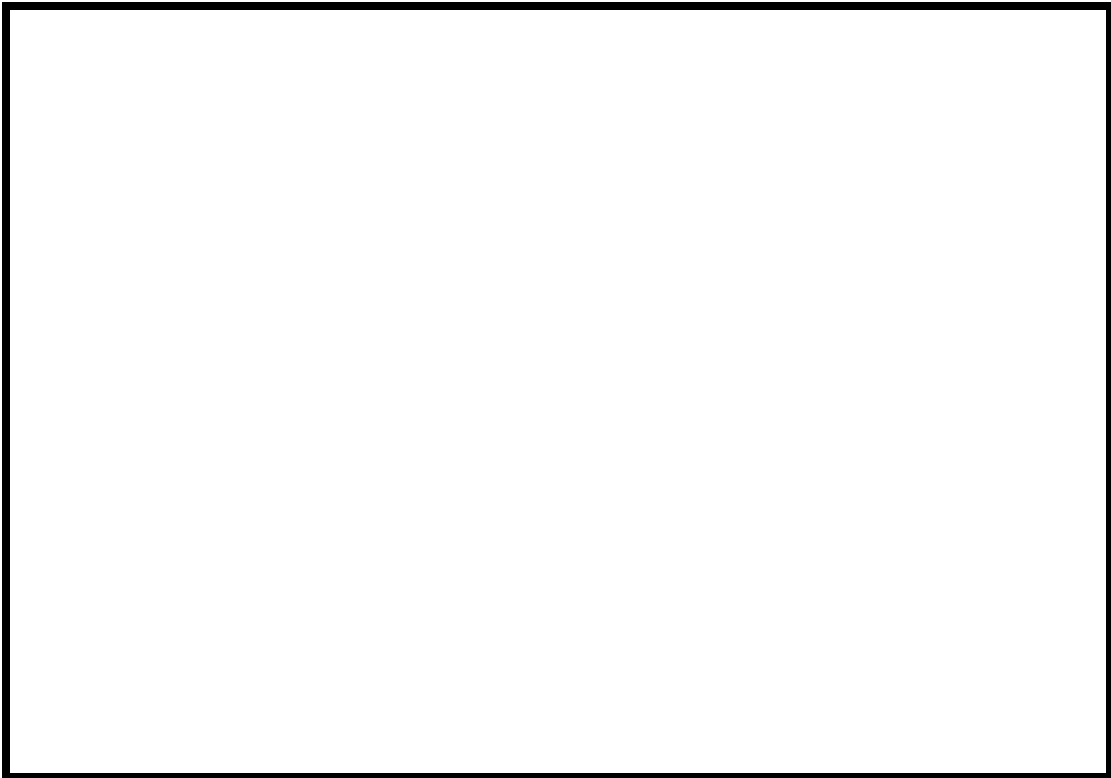
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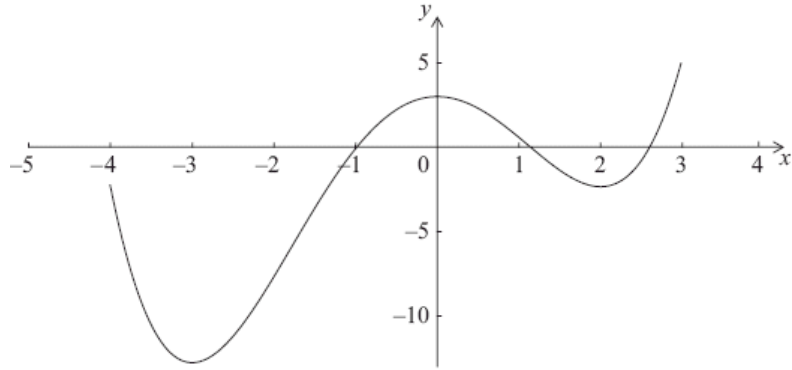
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13. [Maximum mark: 5]

A function  $f$  is defined for  $-4 \leq x \leq 3$ . The graph of  $f$  is given below.



The graph has a local maximum when  $x = 0$ , and local minima when  $x = -3$ ,  $x = 2$ .

Determine whether the following values are **positive**, **negative** or **0**.

	$f(x)$	$f'(x)$
$x = -3$	<b>negative</b>	<b>0</b>
$x = -2$		
$x = -1$		
$x = 0$		
$x = 0.5$		
$x = 2$		

14. [Maximum mark: 6]

Consider  $f(x) = x^2 + \frac{p}{x}$ ,  $x \neq 0$ , where  $p$  is a constant.

(a) Find  $f'(x)$ . [2]

(b) There is a minimum value of  $f(x)$  when  $x = -2$ . Find the value of  $p$ . [4]

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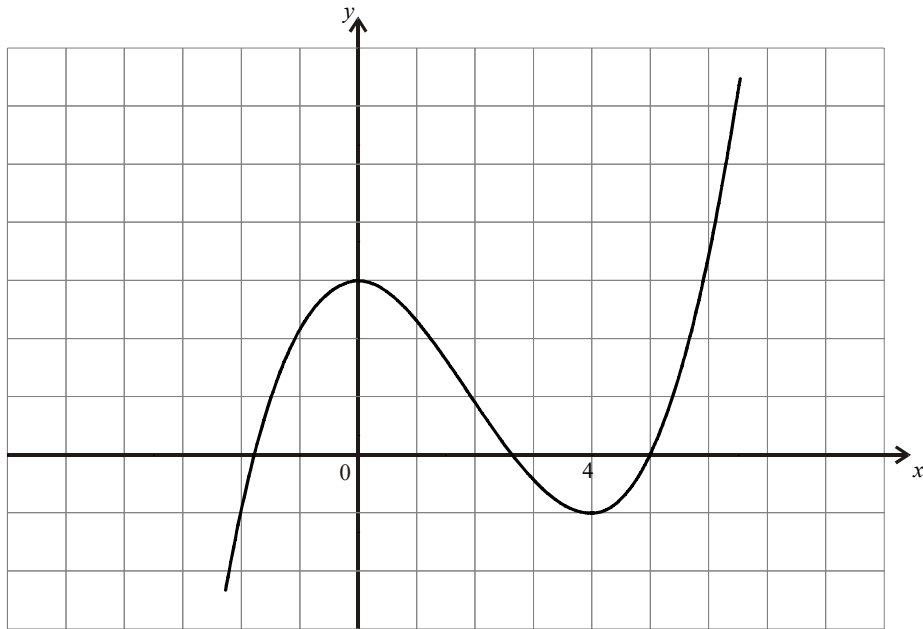
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15. [Maximum mark: 5]

The diagram shows the graph of  $y = f(x)$ .

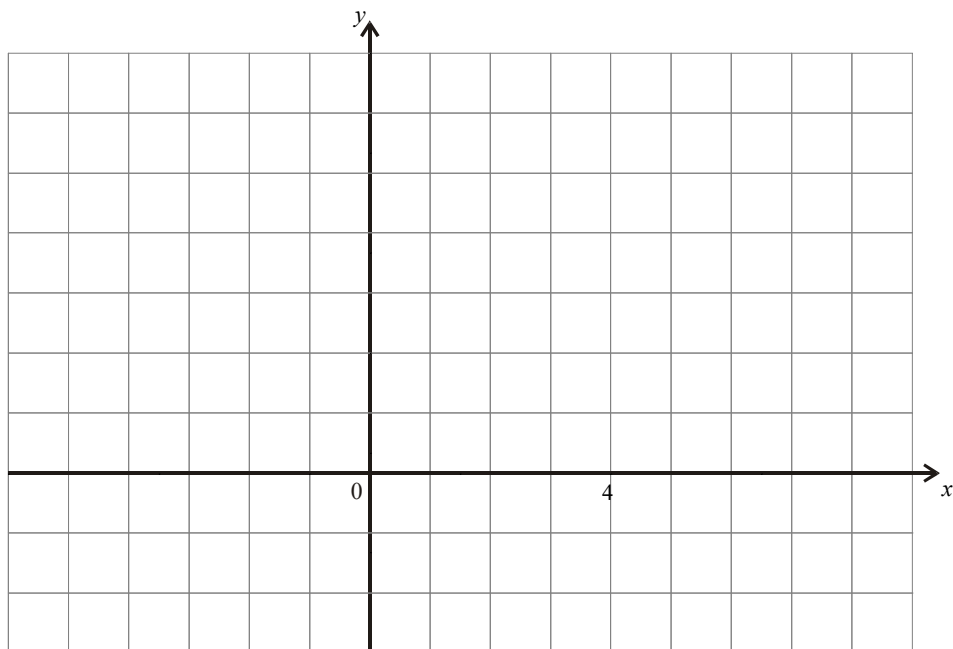


(a) Complete the table below: Use the terms **positive**, **negative** or **0** to note the behavior of  $f'(x)$ :

$x$	$x < 0$	$x = 0$	$0 < x < 4$	$x = 4$	$x > 4$
$f'(x)$					

[3]

(b) Hence, sketch the graph of  $y = f'(x)$  on the grid below.

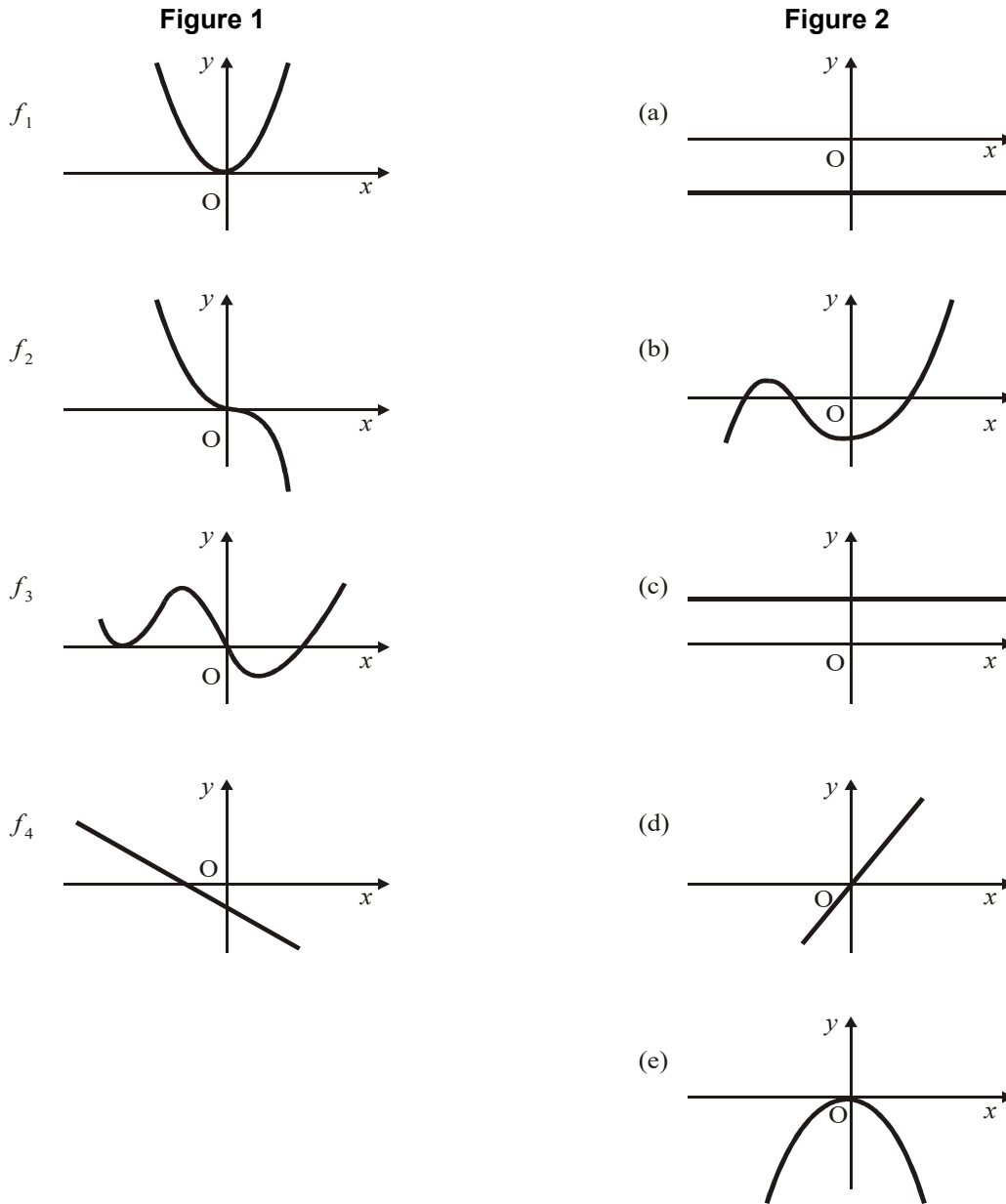


[2]

16. [Maximum mark: 6]

**Figure 1** shows the graphs of the functions  $f_1, f_2, f_3, f_4$ .

**Figure 2** includes the graphs of the derivatives of the functions shown in **Figure 1**, eg the derivative of  $f_1$  is shown in diagram (d).



Complete the table below by matching each function with its derivative.

Function	Derivative diagram
$f_1$	(d)
$f_2$	
$f_3$	
$f_4$	







