NTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 5.16]

AREAS AND VOLUMES

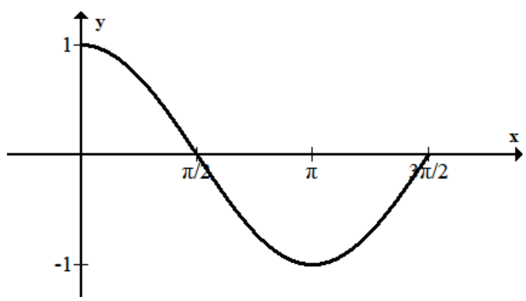
Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

AREAS

1. [Maximum mark: 6]

The following diagram shows part of the graph of $y = \cos x$ for $0 \le x \le \frac{3\pi}{2}$.



(a) Calculate

(i) $\int_0^{\frac{\pi}{2}} \cos x dx$, \`  (ii) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x dx$ ,  \`(iii) $\int_0^{\frac{3\pi}{2}} \cos x dx$ [3]

(b) Write down the area enclosed by the curve and x-axis

(i) between the vertical lines $x = 0$ and $x = \frac{\pi}{2}$

(ii) between the vertical lines $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

(iii) between the vertical lines $x = 0$ and $x = \frac{3\pi}{2}$ [3]

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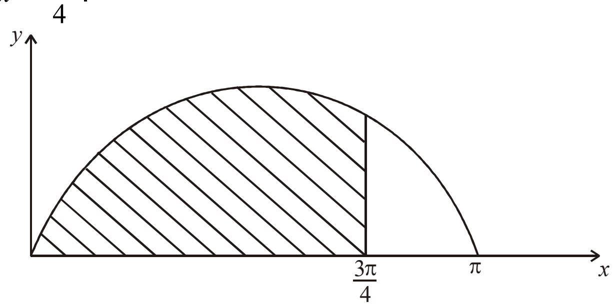
2. [Maximum mark: 6]

The diagram shows part of the curve $y = \sin x$. The shaded region is bounded by the curve and the lines $y = 0$ and $x = \frac{3\pi}{4}$.



Given that $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, find the **exact** area of the shaded region.

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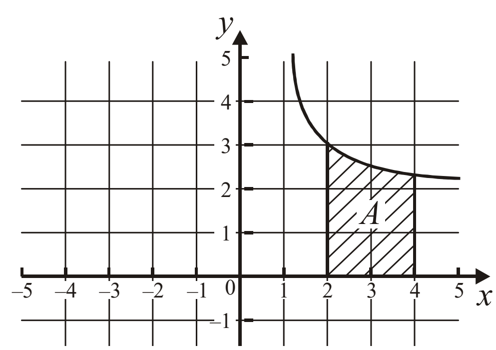
3. [Maximum mark: 6]

Consider the function $f(x) = 2 + \frac{1}{x-1}$. The region enclosed by the graph of $f(x)$, the x-axis and the lines $x = 2$ and $x = 4$, is labelled A, as shown in the diagram below.



Find (i) $\int f(x)\,dx$. (ii) the area of A.

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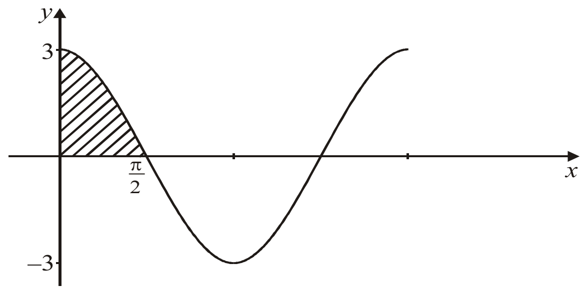
4. [Maximum mark: 4]

The graph represents the function $f : x \mapsto p\cos x$, $p \in \mathbb{N}$.



Write down the value of p and find the area of the shaded region.

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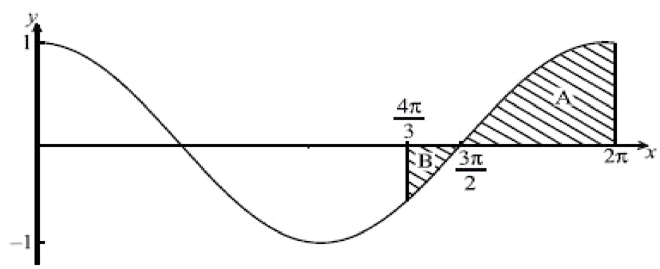
5. [Maximum mark: 6]

The following diagram shows part of the graph of $y = \cos x$ for $0 \le x \le 2\pi$. Regions A and B are shaded.



(a) Calculate the area of A. [2]

(b) Find the total area of the shaded regions in the form $\dfrac{a-\sqrt{b}}{2}$ [4]

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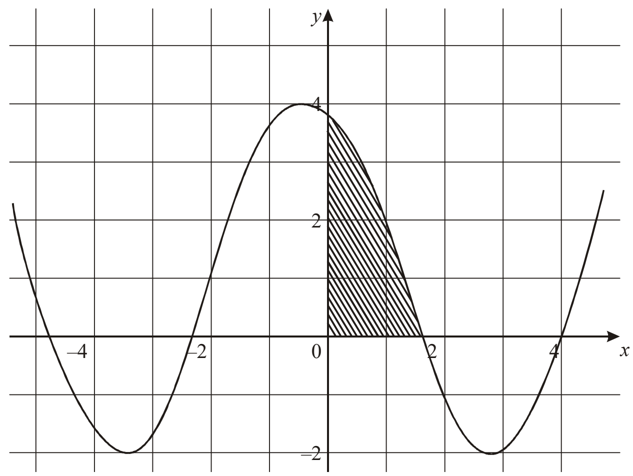
6. [Maximum mark: 7]

(a) Find $\int \left(1+3\sin(x+2)\right)\mathrm{d}x$ [3]

(b) The diagram shows part of the graph of the function $f(x)=1+3\sin(x+2)$.

The area of the shaded region is given by $\int_0^a f(x)\mathrm{d}x$.

(i) Find the value of a. (ii) Find the area of the shaded region. [4]



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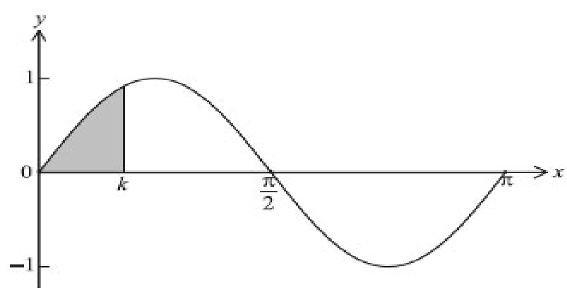
7. [Maximum mark: 6]

The graph of $y=\sin 2x$ is shown below. The area of the shaded region is 0.85. Find k.



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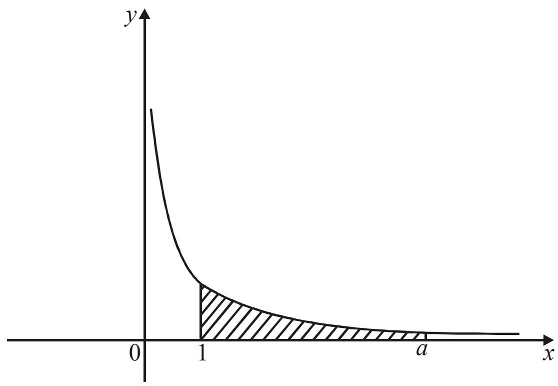
8. [Maximum mark: 4]

The diagram shows part of the graph of $y=\frac{1}{x}$. The shaded region has area 2 units.



Find the **exact** value of a.

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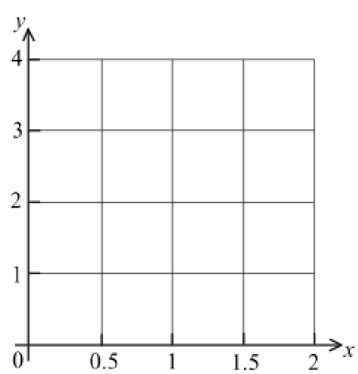
9. [Maximum mark: 6]

For $x \geq \frac{1}{2}$, let $f(x)=x^2\ln(x+1)$ and $g(x)=\sqrt{2x-1}$.

(a) Sketch their graphs of f and g on the grid below. [2]



(b) Let A be the region completely enclosed by the graphs of f and g,

Write down an expression for the area of A and find its value. [4]

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10. [Maximum mark: 4]

Let $f : x \mapsto \frac{\sin x}{x}$, $\pi \le x \le 3\pi$. Find the area enclosed by the graph of *f* and the *x*-axis.

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11. [Maximum mark: 4]

Find the total area of the two regions enclosed by the curve $y = x^3 - 3x^2 - 9x + 27$ and the line $y = x + 3$.

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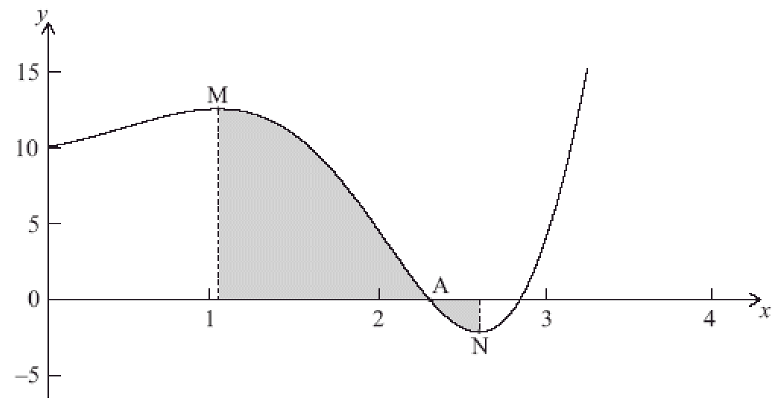
12. [Maximum mark: 15]

Complete the following table

Region enclosed by	Expression for the area	Area
$f(x) = \cos(x^2)$ and $g(x) = e^x$, for $-1.5 \le x \le 0.5$.	$\int_{-1.11}^{0} \cos x^2 - e^x \, dx$	
$y = \sin x$ and $y = x^2 - 2x + 1.5$, for $0 \le x \le \pi$.		0.271
$y = \ln x$ and $y = e^x - e$, for $x > 0$.		
$y = \frac{2}{1+x^2}$ and $y = e^{x/3}$, for $-3 \le x \le 3$.		
$f(x) = 4 - x^2$ and $g(x) = (x+1)\cos x$		
$y = e^{-x} - x + 1$ and the coordinate axes		

13. [Maximum mark: 6]

Let $f(x) = e^x \sin 2x + 10$, for $0 \le x \le 4$. Part of the graph of f is given below.



There is an x-intercept at A, a local max at M with $x = p$, a local min at N with $x = q$.

(a) Write down the x-coordinate of A. [1]

(b) Find the value of (i) p; (ii) q. [2]

(c) Find $\int_p^q f(x)\mathrm{d}x$. Explain why this is not the area of the shaded region. [3]

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15. [Maximum mark: 5]

The function f is defined as $f(x) = e^x \sin x$, where x is in radians.

Let A be the x-intercept corresponding to the smallest **positive** zero of f.

(a) Write down the x-coordinate of the point A. [1]

(b) Let R be the region enclosed by the curve and x-axis, between the origin and A.

Write down an expression for the area of *R* and hence find its value. [4]

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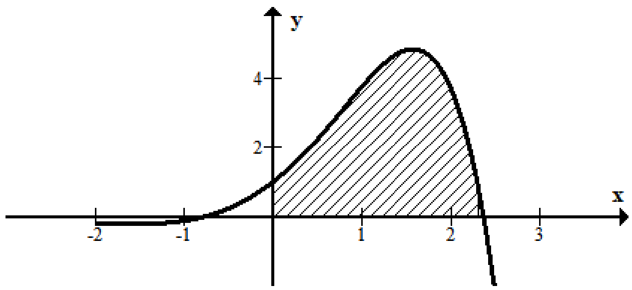
14. [Maximum mark: 6]

Consider the function. $f(x) = \cos x + \sin x$.

(a) Find in terms of π, the smallest **positive** value of x such that $f(x) = 0$. [3]

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \le x \le 3$.



(b) Write down an expression for the area of the shaded region and find its value. [3]

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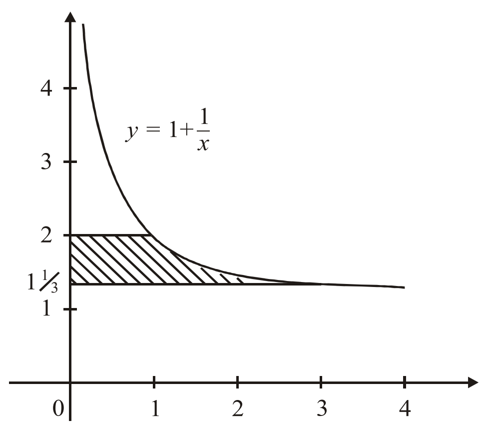
16. [Maximum mark: 6]

The diagram shows the graph of the function $y = 1 + \frac{1}{x}$, $0 < x \le 4$. Find the **exact** value of the area of the shaded region.



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17. [Maximum mark: 8]

Let $f(x) = 2 - \frac{3x}{x^2 - 1}$ and $g(x) = f'(x)$.

(a) Show that $g(x) = \frac{3(x^2+1)}{(x^2-1)^2}$. [4]

(b) Let *A* be the area of the region enclosed by the graph of g and the x-axis, between $x = 0$ and $x = a$, where $a > 0$. Given that $A = 2$, find the value of a. [4]

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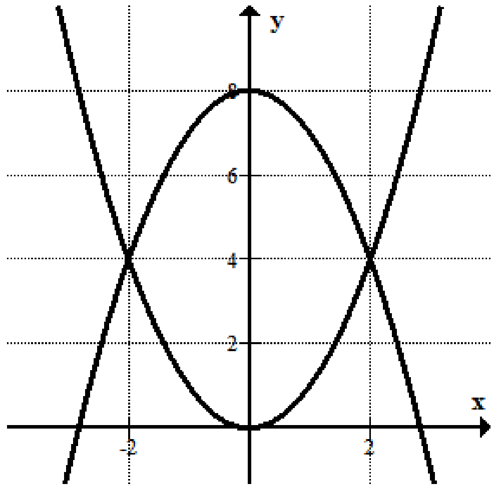
18. [Maximum mark: 8]

Consider the two curves $y = x^2$, $y = 8 - x^2$



(a) Find the area enclosed by the two curves and the ***x*-axis** in the first quadrant

(i) by using the formula $A = \int_a^b y dx$ (ii) by using the formula $A = \int_a^b x dy$

(b) Find the area enclosed by the two curves and the ***y*-axis** in the first quadrant

(i) by using the formula $A = \int_a^b y dx$ (ii) by using the formula $A = \int_a^b x dy$

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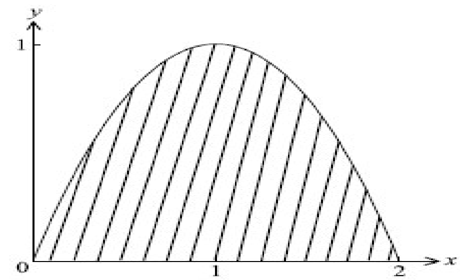
VOLUMES

19. [Maximum mark: 4]

A part of the graph of $y = 2x - x^2$ is given in the diagram below.



The shaded region is revolved through $360°$ about the x-axis.

(a) Write down an expression for this volume of revolution. [2]

(b) Calculate this volume. [2]

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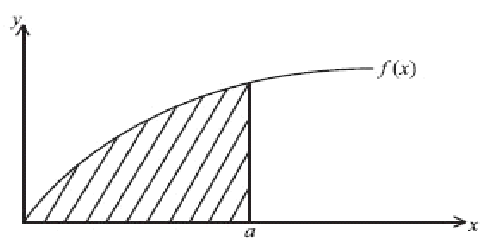
20. [Maximum mark: 5]

The shaded region in the diagram below is bounded by $f(x) = \sqrt{x}$, the line $x = a$ and the x-axis. The shaded region is revolved around the x-axis through $360°$. The volume of the solid formed is 0.845π. Find the value of α



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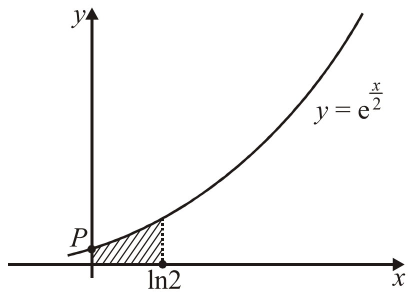
21. [Maximum mark: 7]

The diagram shows part of the graph of $y = e^{\frac{x}{2}}$.



(a) Find the coordinates of the point *P*, where the graph meets the y-axis. [2]

The shaded region between the graph and the x-axis, bounded by $x = 0$ and $x = \ln 2$, is rotated through 360° about the x-axis.

(b) Write down an integral which represents the volume of the solid obtained. [2]

(c) Show that this volume is π. [3]

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22. [Maximum mark: 6]

(a) Find $\int 3\sin^2 x \cos x dx$ [3]

(b) Let $g(x) = \sqrt{3}\sin x(\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis. [3]

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23. [Maximum mark: 4]

Consider the function $f(x)=e^{2x-1}+\frac{5}{2x-1}$, $x \neq \frac{1}{2}$.

The region between the curve and the x-axis between $x=1$ and $x=1.5$ is rotated through 360° about the x-axis. Let *V* be the volume formed.

(a) Write down an expression to represent *V*. [3]

(b) Hence write down the value of *V*. [1]

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24. [Maximum mark: 4]

The graph of $y=\sin(3x)$ for $0 \leq x \leq \frac{\pi}{4}$ is rotated through 2π radians about the x-axis. Find the volume of the solid of revolution formed.

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25. [Maximum mark: 6]

The function f is defined by $f(x)=\frac{\ln x}{x^3}$, $x \geq 1$.

The region enclosed by the x-axis, the graph of f and the line $x=3$ is denoted by *R*.

(a) Find the area of *R*. [3]

(b) Find the volume of the solid of revolution obtained when *R* is rotated through 360° about the x-axis. [3]

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26. [Maximum mark: 4]

The area between the graph of $y = e^x$ and the x-axis from $x = 0$ to $x = k$ ($k > 0$) is rotated through 360° about the x-axis. Find, in terms of k and e, the volume of the solid generated.

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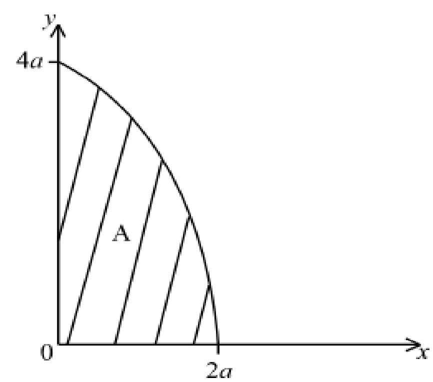
27. [Maximum mark: 6]

The diagram below shows the shaded region *A* which is bounded by the axes and part of the curve $y^2 = 8a(2a - x)$, $a > 0$. Find in terms of a the volume of the solid formed when *A* is rotated through 360° around the x-axis.



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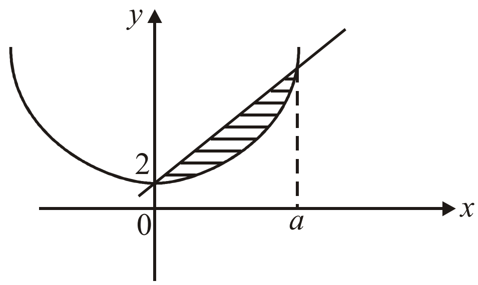
28. [Maximum mark: 10]

The area of the enclosed region shown in the diagram is defined by $y \geq x^2 + 2$ and $y \leq ax + 2$, where $a > 0$.



(a) Find the area of the region. [3]

(b) This region is rotated 360° about the x-axis to form a solid of revolution. Find, in terms of a, the volume of this solid of revolution. [4]

(c) This region is rotated 360° about the y-axis to form a solid of revolution. Write down an expression that represents the volume of this solid of revolution. [3]

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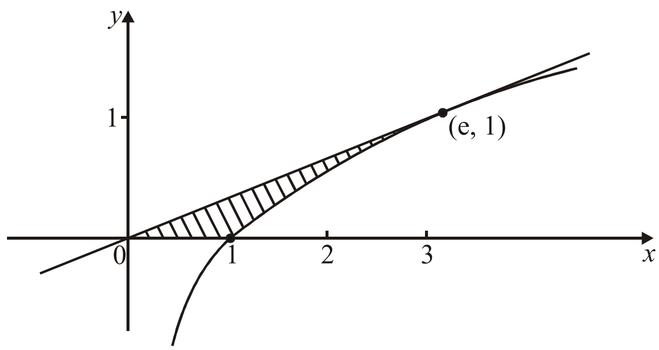
B. Paper 2 questions (LONG)

29. [Maximum mark: 10]

(a) Find the equation of the tangent line to the curve $y = \ln x$ at the point $(\mathrm{e}, 1)$, and verify that the origin is on this line. [4]

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x - x) = \ln x$ [2]

(c) The diagram shows the region enclosed by the curve $y = \ln x$, the tangent line in part (a), and the line $y = 0$.



Use the result of part (b) to show that the area of this region is $\frac{1}{2}e - 1$. [4]

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30. [Maximum mark: 11]

Let $h(x) = \frac{3x-5}{x-2}$, $x \neq 2$.

(a) (i) **Sketch** the graph of h for $-3 \leq x \leq 7$ and $-2 \leq y \leq 8$, including asymptotes.

(ii) Write down the **equations** of the asymptotes. [5]

(b) The expression $\frac{3x-5}{x-2}$ may also be written as $3 + \frac{1}{x-2}$. Use this to find

(i) $\int h(x)\,\mathrm{d}x$.

(ii) the **exact** value of $\int_3^5 h(x)\mathrm{d}x$. [5]

(c) On your sketch, shade the region whose area is represented by $\int_3^5 h(x)\mathrm{d}x$. [1]

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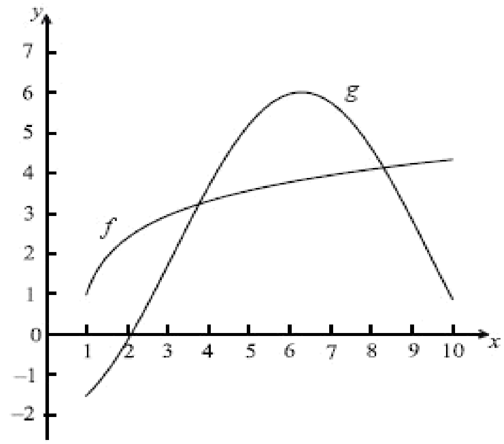
31. [Maximum mark: 14]

The following diagram shows the graphs of $f(x) = \ln(3x - 2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves of f and g.

(i) Find an expression for A;

(ii) Calculate the value of A. [6]

(b) Find (i) $f'(x)$; (ii) $g'(x)$. [4]

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x. [4]

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32. [Maximum mark: 10]

(a) Sketch the graph of $y = \pi \sin x - x$, $-3 \le x \le 3$. Label and number both axes and indicate clearly the approximate positions of the x-intercepts and the local maximum and minimum points. [5]

(b) Find the solution of the equation $\pi \sin x - x = 0$, $x > 0$. [1]

(c) Find the indefinite integral $\int (\pi \sin x - x)\,dx$ and hence, or otherwise, calculate the area of the region enclosed by the graph, the x-axis and the line $x = 1$. [4]

33. [Maximum mark: 13]

Note: Radians are used throughout this question.

(a) (i) Sketch the graph of $y = x^2 \cos x$, for $0 \le x \le 2$ making clear the approximate positions of the positive intercept, the maximum point and the end-points.

(ii) Write down the **approximate** coordinates of the positive x-intercept, the maximum point and the end-points. [7]

(b) Find the **exact value** of the positive x-intercept for $0 \le x \le 2$. [2]

Let R be the region in the first quadrant enclosed by the graph and the x-axis.

(c) (i) Shade *R* on your diagram.

(ii) Write down an integral which represents the area of *R.* [3]

(d) Evaluate the integral in part (c)(ii). [1]

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34. [Maximum mark: 14]

Note: Radians are used throughout this question.

(a) Draw the graph of $y = \pi + x\cos x$, $0 \le x \le 5$, on millimetre square graph paper, using a scale of 2 cm per unit. Make clear

(i) the integer values of x and y on each axis;

(ii) the approximate positions of the x-intercepts and the turning points. [5]

(b) **Without the use of a calculator**, show that π is a solution of the equation $\pi + x\cos x = 0$. [3]

(c) Find another solution of the equation $\pi + x\cos x = 0$ for $0 \le x \le 5$, giving your answer to **six** significant figures. [2]

(d) Let R be the region enclosed by the graph and the axes for $0 \le x \le \pi$. Shade R on your diagram, and write down an integral which represents the area of R. [2]

(e) Evaluate the integral in part (d) to an accuracy of **six** significant figures. [2]

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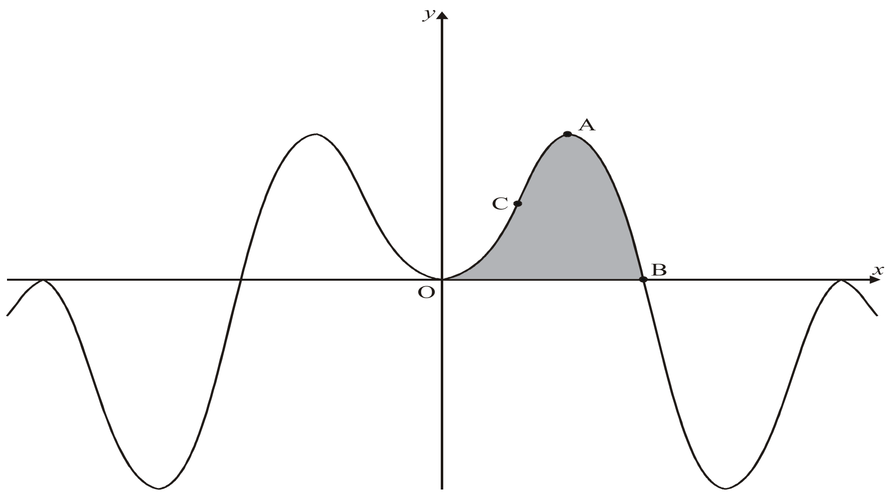
35. [Maximum mark: 20]

Note: Radians are used throughout this question.

The function f is given by $f(x) = (\sin x)^2 \cos x$.

The following diagram shows part of the graph of $y = f(x)$.



The point A is a maximum point, the point B lies on the x-axis, and the point C is a point of inflexion.

(a) Give the period of f [1]

(b) From consideration of the graph of $y = f(x)$, find **to an accuracy of one significant figure** the range of f. [1]

(c) (i) Find $f'(x)$.

(ii) Hence show that at the point A, $\cos x = \sqrt{\frac{1}{3}}$.

(iii) Find the exact maximum value. [9]

(d) Find the exact value of the x-coordinate at the point B. [1]

(e) (i) Find $\int f(x)\,dx$.

(ii) Find the area of the shaded region in the diagram. [4]

(f) Given that $f''(x) = 9(\cos x)^3 - 7\cos x$, find the x-coordinate at the point C. [4]

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36. [Maximum mark: 12]

Consider functions of the form $y = e^{-kx}$.

(a) Show that $\int_0^1 e^{-kx}\,dx = \frac{1}{k}(1 - e^{-k})$. [3]

(b) Let $k = 0.5$

(i) Sketch the graph of $y = e^{-0.5x}$, for $-1 \le x \le 3$, indicating the y-intercept.

(ii) Shade the region enclosed by this graph, x-axis, y-axis and line $x = 1$.

(iii) Find the area of this region. [5]

The region enclosed by the graph of $y = e^{-kx}$, x-axis, y-axis and line $x = 1$ is rotated through 2π about x-axis.

(c) Show that the volume of the solid generated is $\frac{\pi}{2k}(1 - e^{-2k})$ [4]

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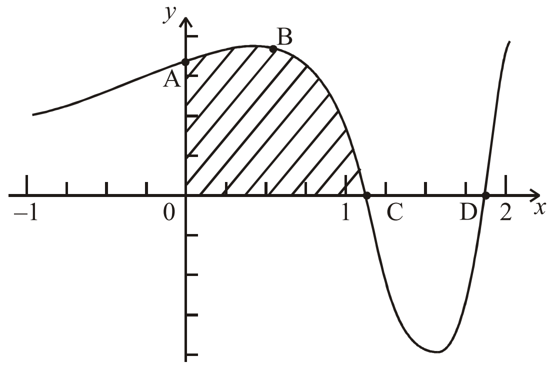
37. [Maximum mark: 15]

The diagram below shows a sketch of the graph of the function $y = \sin(e^x)$ where $-1 \le x \le 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.



(a) Find the coordinates of A. [2]

(b) The coordinates of C may be written as $(\ln k, 0)$. Find the **exact** value of k. [2]

(c) (i) Write down the y-coordinate of B.

(ii) Find $\frac{dy}{dx}$ and hence show that at B, $x = \ln\frac{\pi}{2}$. [6]

(d) Write down the integral which represents the shaded area; evaluate the integral. [5]

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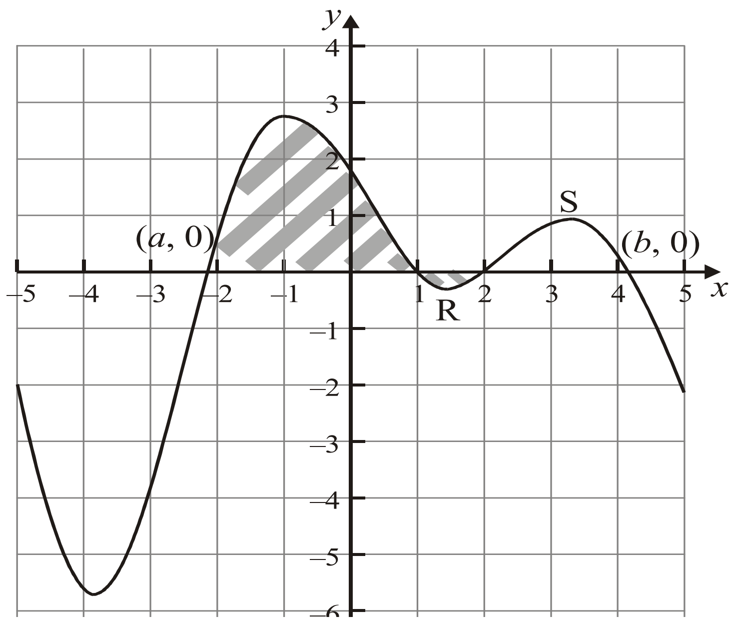
38. [Maximum mark: 11]

Let $h(x) = (x-2)\sin(x-1)$ for $-5 \le x \le 5$. The curve of $h(x)$ is shown below. There is a minimum point at R and a maximum point at S. The curve intersects the x-axis at the points $(a, 0)$, $(1, 0)$, $(2, 0)$ and $(b, 0)$.



(a) Find the exact values of a and b. [2]

The regions between the curve and the x-axis are shaded for $a \le x \le 2$ as shown.

(b) (i) Write down an expression which represents the **total** area shaded.

(ii) Calculate this total area. [5]

(c) (i) The y-coordinate of R is -0.240. Find the y-coordinate of S.

(ii) Hence or otherwise, find the range of values of k for which the equation $(x-2)\sin(x-1) = k$ has **four** distinct solutions. [4]

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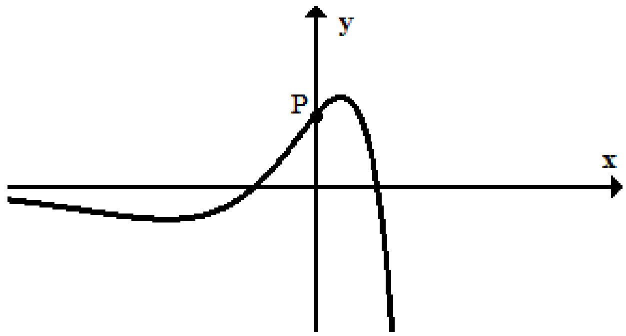
39. [Maximum mark: 13]

Let $f(x)=e^{x}(1-x^{2})$. Part of the graph of $y=f(x)$, for $-6\le x\le 2$, is shown below.



(a) Write down an expression for the area enclosed by the curve and x-axis. [3]

(b) Find the coordinates of the y-intercept P. [1]

(c) Let *L* be the normal to the curve at P. Show that *L* has equation $x+y=1$. [4]

(d) Let *R* be the region enclosed by the curve $y=f(x)$ and the line *L*.

(i) Find an expression for the area of *R*.

(ii) Calculate the area of *R*. [5]

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40. [Maximum mark: 19]

The function f is defined as $f(x)=(2x+1)e^{-x}$, $0 \le x \le 3$. The point P(0, 1) lies on the graph of $f(x)$, and there is a maximum point at Q.

(a) Sketch the graph of $y=f(x)$, labelling the points P and Q. [3]

(b) (i) Show that $f'(x)=(1-2x)e^{-x}$.

(ii) Find the **exact** coordinates of Q. [7]

(c) The equation $f(x)=k$, where $k \in \mathbb{R}$, has two solutions. Write down the range of values of k. [2]

(d) Let R be the point on the curve of f with x-coordinate 3. Find the area of the region enclosed by the curve and the line (PR). [7]

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41. [Maximum mark: 15]

Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.

(a) On the same diagram, sketch the graphs of f and g. [3]

(b) Consider the graph of f. Write down

(i) the x-intercept between $x = 0$ and $x = 3$; (ii) the period; (iii) the amplitude. [4]

(c) Consider the graph of g. Write down

(i) the two x-intercepts; (ii) the equation of the axis of symmetry.

(d) Let *R* be the region enclosed by the graphs of f and g. Find the area of *R*. [6]

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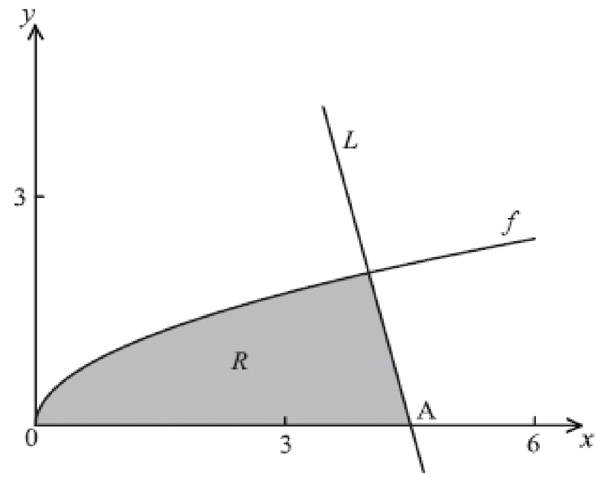
42. [Maximum mark: 12]

Let $f(x)=\sqrt{x}$. Line *L* is the normal to the graph of f at the point (4, 2).

(a) Show that the equation of *L* is $y=-4x+18$. [4]

(b) Point A is the x-intercept of *L*. Find the x-coordinate of A. [2]

In the diagram below, the shaded region *R* is bounded by the x-axis, the graph of f and the line *L*.



(c) Find an expression for the area of *R*. [3]

(d) Find the area *R* [3]

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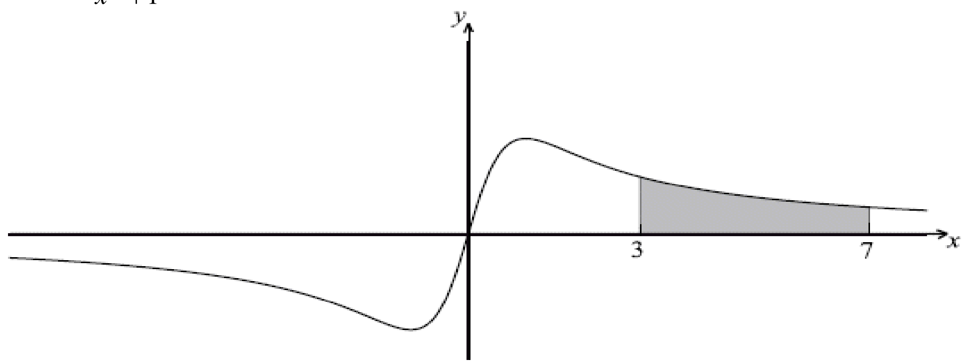
43. [Maximum mark: 14]

Let $f(x)=\dfrac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$. The graph of f is shown below.



(a) Given that $f''(x)=\dfrac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion. [7]

(b) It is given that $\int f(x)\mathrm{d}x=\dfrac{a}{2}\ln(x^2+1)+C$.

(i) Find the area of the shaded region, giving your answer in the form $p\ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$. [7]

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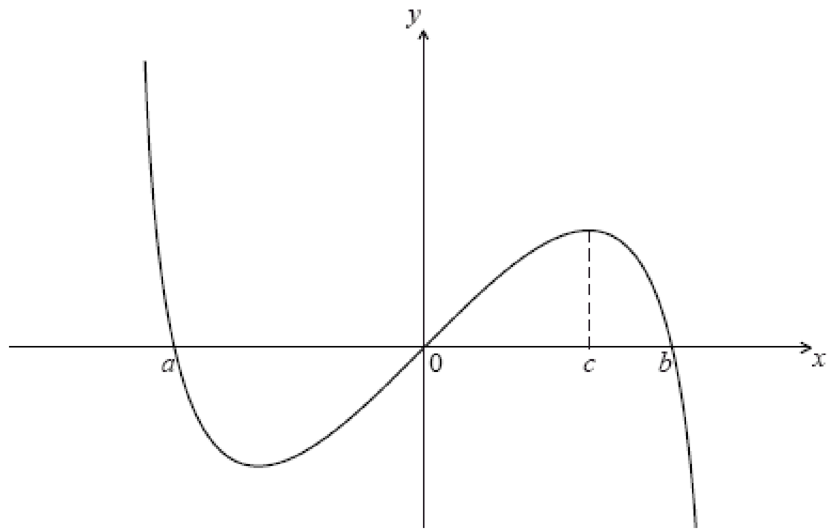
44. [Maximum mark: 9]

Let $f(x) = x\ln(4 - x^2)$, for $-2 \le x \le 2$. The graph of f is shown below.



The graph of f crosses the x-axis at $x = a$, $x = 0$ and $x = b$.

(a) Find the value of a and of b. [3]

The graph of f has a maximum value when $x = c$.

(b) Find the value of c. [2]

(c) Let *R* be the region enclosed by the curve, the x-axis and the line $x = c$, between $x = a$ and $x = c$. Find the area of *R*. [4]

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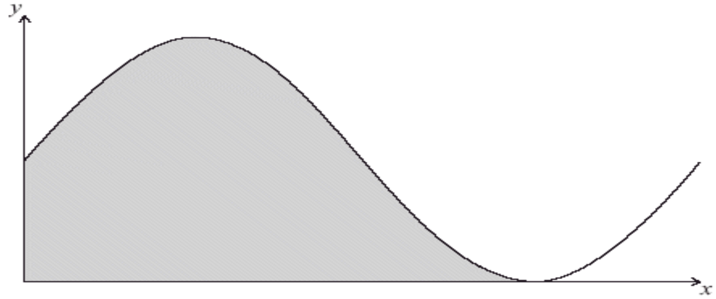
45. [Maximum mark: 17]

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f, the x-axis, and the y-axis.

(a) Solve for $0 \leq x < 2\pi$. (i) $6 + 6\sin x = 6$; (ii) $6 + 6\sin x = 0$. [5]

(b) Write down the exact value of the x-intercept of f, for $0 \leq x < 2\pi$. [1]

(c) The area of the shaded region is k. Find the value of k, in terms of π. [6]

Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g.

(d) Give a full geometric description of this transformation. [2]

(e) Given that $\int_{p}^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \leq p < 2\pi$, write down the two values of p. [3]

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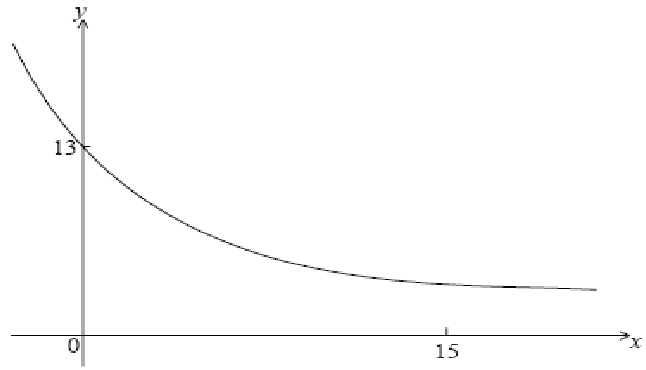
46. [Maximum mark: 17]

Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below. The y-intercept is at (0,13).



(a) Show that $A = 10$. [2]

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k. [3]

(c) (i) Using your value of k, find $f'(x)$.

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f. [6]

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of f and g. [6]

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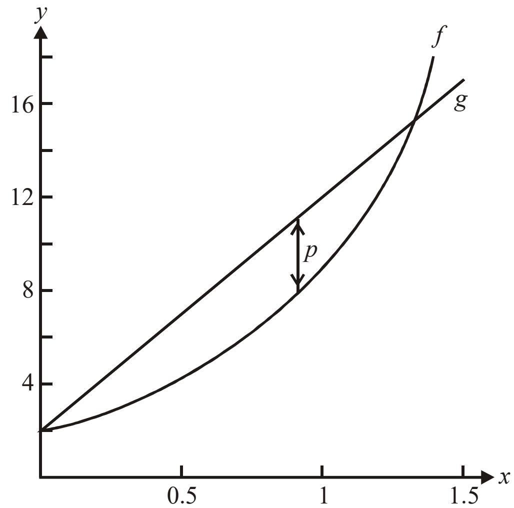
47. [Maximum mark: 15]

The diagram below shows the graphs of $f(x)=1+e^{2x}$, $g(x)=10x+2$, $0 \le x \le 1.5$.



(a) (i) The graphs of f and g intersect at $x=0$ and $x=a$. Find the value of a.

(ii) Find an expression for the vertical distance p between the two graphs.

(iii) Given that p has a maximum value for $0 \le x \le a$, find the value of x at which this occurs.

(iv) Hence find the maximum value of p. [8]

Let R be the region enclosed by enclosed by the graphs of f and g.

(b) Find the area of the region R. [3]

(c) Find the volume of the solid generated when R Is rotated through 2π in x-axis. [4]

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48. [Maximum mark: 14]

(a) On the same axes sketch the graphs of the functions, $f(x)$ and $g(x)$, where

$f(x) = 4 - (1 - x)^2$, for $-2 \le x \le 4$,

$g(x) = \ln(x + 3) - 2$, for $-3 \le x \le 5$. [2]

(b) (i) Write down the equation of any vertical asymptotes.

(ii) State the x-intercept and y-intercept of $g(x)$. [3]

(c) Find the values of x for which $f(x) = g(x)$. [2]

(d) Let *A* be the region where $f(x) \ge g(x)$ and $x \ge 0$.

(i) On your graph shade the region *A*.

(ii) Write down an integral that represents the area of *A*.

(iii) Evaluate this integral. [4]

(e) In the region *A* find the maximum vertical distance d between $f(x)$ and $g(x)$. [3]

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49. [Maximum mark: 21]

The function f is defined on the domain $x \geq 1$ by $f(x) = \frac{\ln x}{x}$.

(a) (i) Show, by considering the first and second derivatives of f, that there is one *l.* maximum point on the graph of f.

(ii) State the **exact** coordinates of this point. [9]

(b) The graph of f has a point of inflexion at P. Find the x-coordinate of P. [3]

Let *R* be the region enclosed by the graph of f, the x-axis and the line $x = 5$.

(c) Find the **exact** value of the area of *R*. [6]

(d) The region R is rotated through an angle 2π about the x-axis. Find the volume of the solid of revolution generated. [3]

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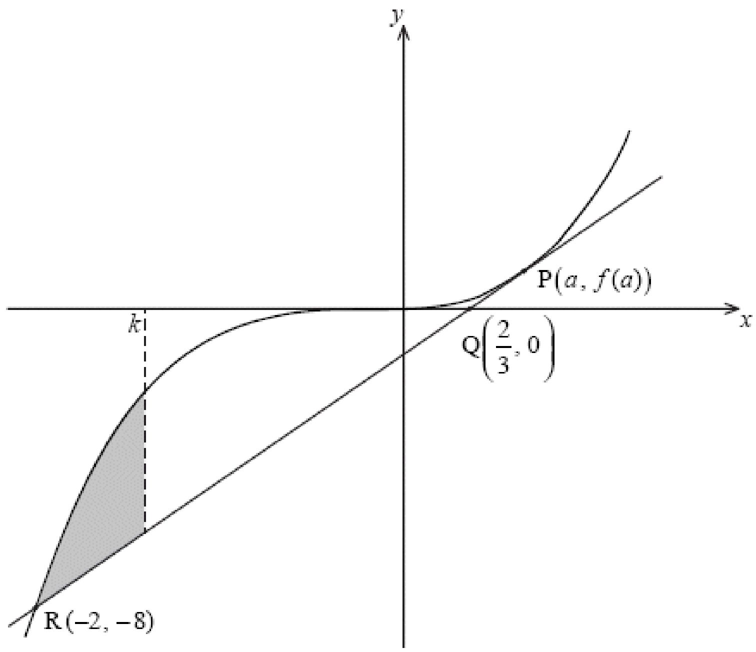
50. [Maximum mark: 16]

Let $f(x) = x^3$.

The point P $(a, f(a))$, where $a > 0$, lies on the graph of f. The tangent at P crosses the x-axis at the point $Q\left(\frac{2}{3}, 0\right)$ and intersects the graph of f at the point $R(-2, -8)$ as shown in the diagram below.



(a) (i) Show that the gradient of [PQ] is $\dfrac{a^3}{a - \frac{2}{3}}$.

(ii) Find $f'(x)$ and hence $f'(a)$ in terms of a.

(iii) Hence show that $a = 1$. [7]

The equation of the tangent at P is $y = 3x - 2$.

Let *T* be the region enclosed by the graph of f, the tangent [PR] and the line $x = k$, between $x = -2$ and $x = k$ where $-2 < k < 1$. This is shown in the diagram above.

(b) Given that the area of *T* is $2k + 4$, show that k satisfies the equation $k^4 - 6k^2 + 8 = 0$. [9]

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51. [Maximum mark: 20]

Consider the three curves $y = e^x$, $y = 4$, $y = 20 - 4x$

(a) Sketch a graph and shade the region R in the first quadrant enclosed by the three curves and the x-axis. Indicate any intercepts and points of intersection. [5]

(b) Find the **exact value** of the area of the region R. [5]

(c) Find the **exact value** of the volume of the solid generated when the region R is rotated 2π radians in x-axis [5]

(d) Write down an expression for the volume of the solid generated when the region R is rotated 2π radians in y-axis and hence find its value. [5]

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