

INTERNATIONAL BACCALAUREATE
Mathematics: applications and interpretation
MAI

EXERCISES [MAI 5.14-5.15]
MORE INTEGRALS – SUBSTITUTION

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A. Paper 1 questions (SHORT)

INDEFINITE INTEGRALS

1. [Maximum mark: 18]

Find the following integrals

$$\int \left(\cos x - \sin x + e^x + \frac{1}{x} \right) dx$$

$$\int \left(5 \sin x - 3 \cos x - 7e^x + \frac{1}{x} \right) dx$$

$$\int (x^{-3} + x^{-2} + x^{-1} + 3) dx$$

$$\int (4x^{-3} - 12x^{-2} + 6x^{-1} + 3) dx$$

$$\int (x^{\frac{1}{2}} + x^{\frac{1}{3}}) dx$$

$$\int (5x^{\frac{1}{2}} - 3x^{\frac{1}{3}}) dx$$

$$\int (x^{\frac{2}{5}} + x^{\frac{3}{7}}) dx$$

$$\int (7x^{\frac{2}{5}} + 10x^{\frac{3}{7}}) dx$$

$$\int \left(\cos x + \frac{1}{\cos^2 x} \right) dx$$

2. [Maximum mark: 27]

Find the following integrals (modify the expressions first, if necessary).

$$\int \left(\frac{4}{x^3} - \frac{12}{x^2} + \frac{6}{x} + 3 \right) dx =$$

$$\int \left(\frac{1}{4x^3} - \frac{2}{3x^2} - \frac{5}{7x} + 3 \right) dx =$$

$$\int 4x^2 \left(x - \frac{1}{x^3} \right) dx =$$

$$\int (5\sqrt{x} - 3\sqrt[3]{x}) dx =$$

$$\int (5\sqrt{x^3} - 3\sqrt[3]{x^2}) dx =$$

$$\int 5x(\sqrt{x} + x^3) dx =$$

$$\int \frac{1+x}{x} dx =$$

$$\int \frac{2x^7 + 5x + 4}{3x^2} dx =$$

$$\int \frac{2x \cos^2 x + 1}{\cos^2 x} dx =$$

3. [Maximum mark: 30]

Find the following integrals

$\int \sin(5x + 3)dx$
$\int \cos(5x + 3)dx$
$\int e^{5x+3} dx$
$\int (5x + 3)^3 dx$
$\int \sin 5x dx$
$\int \cos 5x dx$
$\int e^{5x} dx$
$\int \sin(x + 5)dx$
$\int \cos(x + 5)dx$
$\int e^{x+5} dx$
$\int (x + 5)^3 dx$
$\int \sin(1 - x)xdx$
$\int \cos(1 - x)dx$
$\int e^{1-x} dx$
$\int (1 - x)^3 dx$

4. [Maximum mark: 21]

Find the following integrals

$\int \frac{1}{5x+3} dx$
$\int \frac{1}{3-5x} dx$
$\int \frac{1}{x-3} dx$
$\int \frac{1}{3-x} dx$
$\int \frac{8}{2x+1} dx$
$\int \frac{8}{1-2x} dx$
$\int \frac{a}{bx+c} dx$

5. [Maximum mark: 21]

Find the following integrals

$\int \frac{1}{(5x+3)^2} dx$
$\int \frac{1}{(3-5x)^2} dx$
$\int \frac{1}{(x-3)^2} dx$
$\int \frac{1}{(3-x)^2} dx$
$\int \frac{8}{(2x+1)^2} dx$
$\int \frac{8}{(1-2x)^2} dx$
$\int \frac{a}{(bx+c)^2} dx$

6. [Maximum mark: 6]

Let $f(x) = \sqrt{x^3}$. Find

(a) $f'(x)$; [3]

(b) $\int f(x)dx$. [3]

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7. [Maximum mark: 6]

Find

(a) $\int \sin(3x+7) dx$; [2]

(b) $\int e^{-4x} dx$. [2]

(c) $\int \cos(1-x)dx$ [2]

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8. [Maximum mark: 6]

The function f is given by $f(x) = 2 \sin(5x - 3)$.

(a) Find $f''(x)$. [4]

(b) Write down $\int f(x)dx$. [2]

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9. [Maximum mark: 6]

Let $f(x) = (3x + 4)^5$. Find (i) $f'(x)$; (ii) $\int f(x)dx$.

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10. [Maximum mark: 6]

Given that $f(x) = (2x + 5)^3$ find (i) $f'(x)$; (ii) $\int f(x)dx$.

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11. [Maximum mark: 4]

If $f'(x) = \cos x$, and $f\left(\frac{\pi}{2}\right) = -2$, find $f(x)$.

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12. [Maximum mark: 7]

A gradient function is given by $\frac{dy}{dx} = 10e^{2x} - 5$. When $x = 0$, $y = 8$. Find the value of y when $x = 1$.

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13. [Maximum mark: 6]

The derivative of the function f is given by $f'(x) = \frac{1}{1+x} - 0.5 \sin x$, for $x \neq -1$.

The graph of f passes through the point $(0, 2)$. Find an expression for $f(x)$.

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14. [Maximum mark: 6]

The derivative of the function f is given by $f'(x) = e^{-2x} + \frac{1}{1-x}$, $x < 1$.

The graph of $y = f(x)$ passes through the point $(0, 4)$. Find an expression for $f(x)$.

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15. [Maximum mark: 6]

The graph of the function $y = f(x)$ passes through the point $\left(\frac{3}{2}, 4\right)$. The gradient function of f is given as $f'(x) = \sin(2x - 3)$. Find $f(x)$.

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16. [Maximum mark: 8]

Find (i) Find $\int \frac{1}{2x+3} dx$. (ii) $\int \frac{1}{(2x+3)^2} dx$ (iii) $\int \frac{1}{(2x+3)^3} dx$

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SUBSTITUTION

17. [Maximum mark: 6]

Find $\int \frac{6}{2x-3} dx$ (i) directly by inspection (ii) by using the substitution $u = 2x - 3$

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18. [Maximum mark: 5]

Find $\int \frac{6x}{2x^2-3} dx$, by using the substitution $u = 2x^2 - 3$

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20. [Maximum mark: 8]

Find (i) $\int 3x \sin(x^2 + 1) dx$ (ii) $\int 3x^2 \cos(x^3 + 1) dx$

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21. [Maximum mark: 5]

Find $\int \frac{3x^2}{x^3 + 1} dx$

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22. [Maximum mark: 5]

Find $\int \frac{(\ln x)^5}{x} dx$

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23. [Maximum mark: 5]

Find $\int \frac{\tan^5 x}{\cos^2 x} dx$

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DEFINITE INTEGRALS

24. [Maximum mark: 20]

Calculate the following definite integrals

$\int_0^1 (e^x + 2) dx$
$\int_0^\pi (\sin x + \cos x) dx$
$\int_1^e \frac{7}{x} dx$
$\int_0^1 e^{2x+3} dx$
$\int_0^4 \frac{1}{x+1} dx$

25. [Maximum mark: 6]

Let $f(x) = x \ln x - x$

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|--|-----|
| (a) Find $f'(x)$ | [3] |
| (b) Hence find the exact value of $\int_1^3 \ln x dx$. | [3] |

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26. [Maximum mark: 6]

(a) Find $\int \frac{1}{2x+3} dx$. [2]

(b) Given that $\int_0^3 \frac{1}{2x+3} dx = \ln \sqrt{P}$, find the value of P . [4]

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27. [Maximum mark: 6]

Given $\int_3^k \frac{1}{x-2} dx = \ln 7$, find the value of k .

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28. [Maximum mark: 10]

Let $\int_5^7 f(x)dx = 8$. Find the values of the following expressions

$\int_7^5 \frac{f(x)}{4} dx$
$\int_5^8 f(x)dx - \int_7^8 f(x)dx$
$3\int_5^6 f(x)dx + \int_6^7 3f(x)dx$
$\int_8^{10} f(x-3)dx$
$\int_{2.5}^{3.5} f(2x)dx$

29. [Maximum mark: 7]

Let f be a function such that $\int_0^3 f(x) dx = 8$.

(a) Given that $\int_c^d f(x-2)dx = 8$, write down the value of c and of d . [2]

(b) Given that $\int_a^b f(2x)dx = 4$, write down the value of a and of b . [2]

(c) Find the **exact** value of $\int_0^3 (f(x) + e^x) dx$. [3]

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B. Paper 2 questions (LONG)

30. [Maximum mark: 12]

Find (i) $\int 2\sqrt{x+3}dx$ (ii) $\int 2x\sqrt{x^2+3}dx$ (iii) $\int 2x^2\sqrt{x^3+3}dx$

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33. [Maximum mark: 12]

Use the substitution $u = \ln x$ to find the integrals

(i) $\int \frac{(\ln x)^2}{x} dx$ (ii) $\int \frac{\sqrt{\ln x}}{x} dx$ (iii) $\int \frac{1}{x(\ln x)^2} dx$ (iv) $\int \frac{\ln x}{x} dx$

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