INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 5.1]

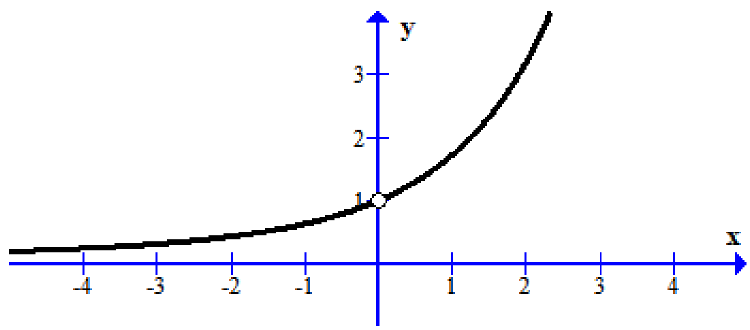
THE CONCEPT OF THE LIMIT

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 5]

Part of the graph of $f(x)=\dfrac{e^x-1}{x}$ is shown below. The function is not defined at $x=0$.



(a) Complete the values of f, **correct to 4 decimal places**, on the tables below:

For positive values of x near 0 (i.e. 0^+)	
x	$f(x)$
0.1	
0.01	
0.001	

For negative values of x near 0^-(i.e. 0^-)	
x	$f(x)$
– 0.1	
– 0.01	
– 0.001	

[4]

(b) Deduce the value of the limit: $\lim_{x\to 0}\dfrac{e^x-1}{x}$ [1]

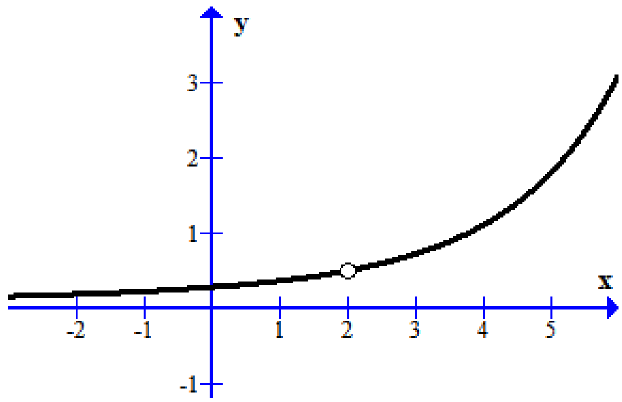
....

....

....

2. [Maximum mark: 7]

Part of the graph of $f(x)=\dfrac{e^{x-2}-x+1}{(x-2)^2}$ is shown below:



(a) Write down, **correct to 4 s.f.** , the value of $f(0)$ [1]

(b) Explain why $f(2)$ is not defined. [1]

(c) Complete the values of f , **correct to 4 s.f.** on the tables below:

For values of x near 2^+:	
x	$f(x)$
2.1	
2.01	
2.001	

For values of x near 2^-	
x	$f(x)$
1.9	
1.99	
1.999	

[4]

(d) Deduce the value of the limit: $\lim\limits_{x\to 2}\dfrac{e^{x-2}-x+1}{(x-2)^2}$ [1]

....

....

....

....

....

....

....

....

....

3. [Maximum mark: 7]

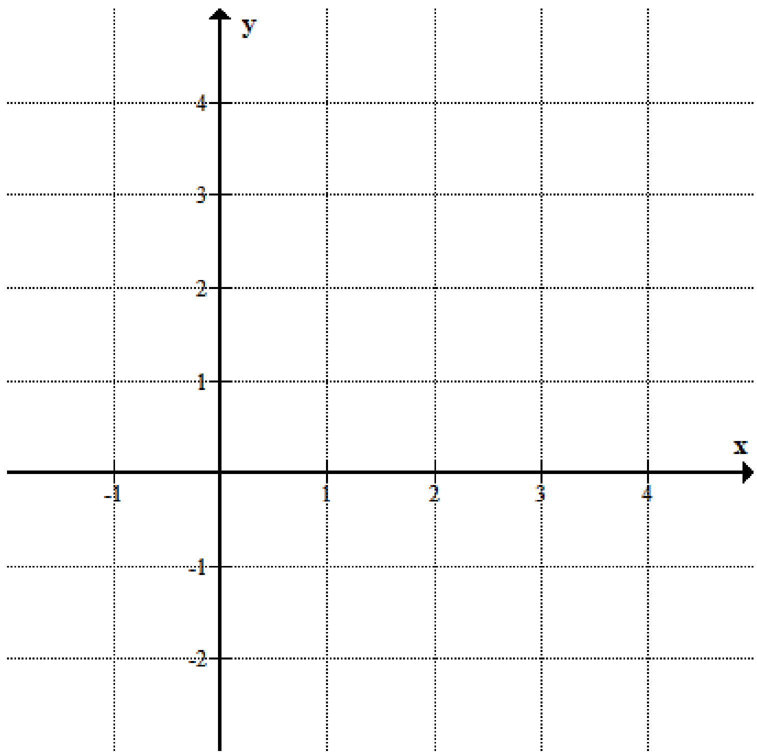
Let $P = \dfrac{4Q - 4\ln(1+Q)}{Q^2}$, $Q \neq 0$, $-0.5 \leq Q \leq 4$,

(a) Write down the values of P for

(i) $Q = -0.5$ (ii) $Q = 1$, (iii) $Q = 4$. [3]

(b) On the following diagram, sketch the graph of P vs Q, for $-0.5 \leq Q \leq 4$ [2]

(c) P is not defined for $Q = 0$. By investigating the values of P corresponding to values of Q near zero, deduce the value of $\lim\limits_{Q \to 0} P$. [2]



....

....

....

....

....

....

....

....

4. [Maximum mark: 8]

Let $f(x)=\dfrac{3x+2}{x+5}$

(a) Complete the values of f , **correct to 6 s.f.** on the table below:

x	$f(x)$
100	
1 000	
1 000 000	

[2]

(b) Deduce the value of the limit: $\lim\limits_{x\to\infty}\dfrac{3x+2}{x+5}$ [1]

(c) Complete the values of f , **correct to 6 s.f.** on the table below:

x	$f(x)$
–100	
–1 000	
–1 000 000	

[2]

(d) Deduce the value of the limit: $\lim\limits_{x\to-\infty}\dfrac{3x+2}{x+5}$. [1]

(e) By using a similar rationale deduce the value of the limits: $\lim\limits_{x\to\pm\infty}\dfrac{3x+2}{2x+5}$. [2]

....

....

....

....

....

....

....

....

....

....

5. [Maximum mark: 7]

(a) By observing the graph of the function $f(x)=\dfrac{x+3}{x-2}$ on your GDC, or otherwise find the values of the following limits

(i) $\lim\limits_{x\to 3}\dfrac{x+3}{x-2}$ (ii) $\lim\limits_{x\to +\infty}\dfrac{x+3}{x-2}$ (iii) $\lim\limits_{x\to -\infty}\dfrac{x+3}{x-2}$ [3]

(b) Investigate whether each of the following **side** limits is +∞ or -∞:

(i) $\lim\limits_{x\to 2^+}\dfrac{x+3}{x-2}$ (ii) $\lim\limits_{x\to 2^-}\dfrac{x+3}{x-2}$

(iii) $\lim\limits_{x\to 2^+}\dfrac{x-3}{x-2}$ (iv) $\lim\limits_{x\to 2^-}\dfrac{x-3}{x-2}$ [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

6. [Maximum mark: 8]

The gradient of the curve $f(x)=x^2$ at point $x=a$ is defined by the limit

$$m_a = \lim_{h\to 0}\frac{(a+h)^2-a^2}{h}$$

For example, the gradient at $x=1$ is $m_1=2$ since $\lim_{h\to 0}\frac{(1+h)^2-1^1}{h}=2$.

(a) By using the graph mode on your GDC, find

(i) $\lim_{h\to 0}\frac{(2+h)^2-2^2}{h}$ and **hence** the gradient m_2.

(ii) $\lim_{h\to 0}\frac{(3+h)^2-3^2}{h}$ and **hence** the gradient m_3. [4]

(b) Find the gradient of the curve

(i) at $x=5$. (ii) $x=-5$ [3]

(c) Deduce the value of the gradient m_a in terms of a in general. [1]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

7. [Maximum mark: 8]

The gradient of the curve $f(x) = \ln x$ at point $x = a$ is defined by the limit

$$m_a = \lim_{h \to 0} \frac{\ln(a+h) - \ln a}{h}$$

For example, the gradient at $x = 1$ is $m_1 = 1$ since $\lim_{h \to 0} \frac{\ln(1+h) - \ln 1}{h} = 1$.

(a) By using the graph mode on your GDC, find

(i) $\lim_{h \to 0} \frac{\ln(2+h) - \ln 2}{h}$ and **hence** the gradient m_2.

(ii) $\lim_{h \to 0} \frac{\ln(5+h) - \ln 5}{h}$ and **hence** the gradient m_5. [4]

(b) Find the gradient m_{10} of the curve at $x = 10$. [2]

(c) Deduce the value of the gradient m_a in terms of a in general. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

8. [Maximum mark: 8]

(a) Write down the value of e^2 correct to **5 s.f.** [1]

(b) Write down, correct to **5 s.f.**, the values of the expression

$$\left(1+\frac{2}{n}\right)^n$$

for the values of n shown on the following table:

n	$\left(1+\frac{2}{n}\right)^n$
1	
10	
100	
1 000	
1 000 000	

[3]

(c) **Hence**, guess the value of $\lim_{n\to+\infty}\left(1+\frac{2}{n}\right)^n$ [1]

(d) By following a similar rationale guess the value of $\lim_{n\to+\infty}\left(1+\frac{3}{n}\right)^n$ [3]

....

....

....

....

....

....

....

....

....

....

....

....

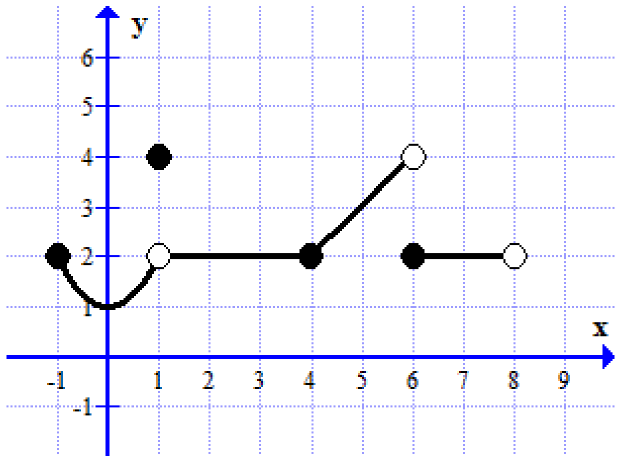
....

....

B. Paper 2 questions (LONG)

9. [Maximum mark: 15]

The graph of the function f is shown below



(a) Write down the domain and the range of the function.

Domain of f :	Range of f :

[2]

(b) Complete the following table of values (if there exist)

$f(-1) =$	$f(1) =$	$f(2) =$	$f(4) =$
$f(5) =$	$f(6) =$	$f(7) =$	$f(8) =$

[4]

(c) Complete the following table of limits (if there exist)

$\lim_{x\to 1^-} f(x) =$	$\lim_{x\to 1^+} f(x) =$	$\lim_{x\to 1} f(x) =$
$\lim_{x\to 4^-} f(x) =$	$\lim_{x\to 4^+} f(x) =$	$\lim_{x\to 4} f(x) =$
$\lim_{x\to 6^-} f(x) =$	$\lim_{x\to 6^+} f(x) =$	$\lim_{x\to 6} f(x) =$

[9]

10. [Maximum mark: 14]

Let

$$f(x)=\begin{cases} x^2+1 & -1\le x<1 \\ 4 & x=1 \\ 2 & 1<x\le 4 \\ x-2 & 4<x<6 \\ 2 & 6\le x<8 \end{cases}$$

(a) Write down the domain of the function.

Domain of f :

[2]

(b) Complete the following table of values (if there exist)

$f(-1)=$	$f(1)=$	$f(2)\ =$	$f(4)=$
$f(5)=$	$f(6)=$	$f(7)\ =$	$f(8)=$

[4]

(c) Complete the following table of limits (if there exist)

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(x^2+1)=2$
$\lim_{x\to 1^+} f(x)=$
$\lim_{x\to 1} f(x)=$
$\lim_{x\to 4^-} f(x)=$
$\lim_{x\to 4^+} f(x)=$
$\lim_{x\to 4} f(x)=$
$\lim_{x\to 6^-} f(x)=$
$\lim_{x\to 6^+} f(x)=$
$\lim_{x\to 6} f(x)=$

[8]