

INTERNATIONAL BACCALAUREATE  
**Mathematics: applications and interpretation**

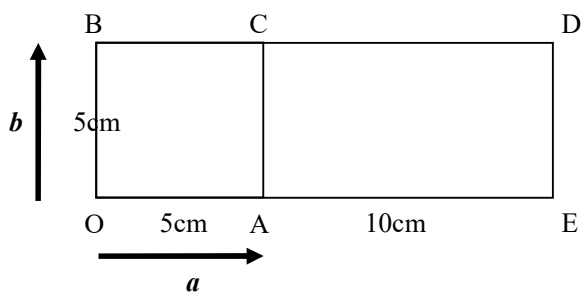
**MAI**

**EXERCISES [MAI 3.9-3.11]**  
**VECTORS – DOT PRODUCT**  
*Compiled by Christos Nikolaidis*

**A. Paper 1 questions (SHORT)**

1. [Maximum mark: 12]

In the following diagram OBCA is a square of side 5cm, while ACDE is a rectangle of length 10cm and width 5cm. Let  $\mathbf{a} = \overrightarrow{OA}$  and  $\mathbf{b} = \overrightarrow{OB}$



(a) Determine whether the following expressions in terms of the vectors  $\mathbf{a}$  and  $\mathbf{b}$  are true or false

$\overrightarrow{AC} = \mathbf{b}$	<b>true</b>	$\overrightarrow{BC} = \mathbf{a}$		$\overrightarrow{BD} = 3\mathbf{a}$	
$\overrightarrow{DE} = \mathbf{b}$		$\overrightarrow{BC} = \mathbf{b}$		$\overrightarrow{CD} = 2\mathbf{b}$	
$\overrightarrow{ED} = \mathbf{a}$		$ \overrightarrow{BC}  =  \mathbf{b} $		$\overrightarrow{CD} = 2\mathbf{a}$	

[4]

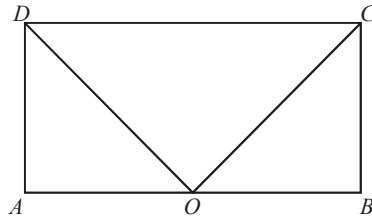
(b) Express the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$

$\overrightarrow{OC} =$	$\overrightarrow{AB} =$
$\overrightarrow{OD} =$	$\overrightarrow{CE} =$
$\overrightarrow{AD} =$	$\overrightarrow{BE} =$
$\overrightarrow{BA} =$	$\overrightarrow{EC} =$

[8]

2. [Maximum mark: 6]

$ABCD$  is a rectangle and  $O$  is the midpoint of  $[AB]$ .



Express each of the following vectors in terms of  $\vec{OC}$  and  $\vec{OD}$

- (a)  $\vec{CD}$       (b)  $\vec{OA}$       (c)  $\vec{AD}$

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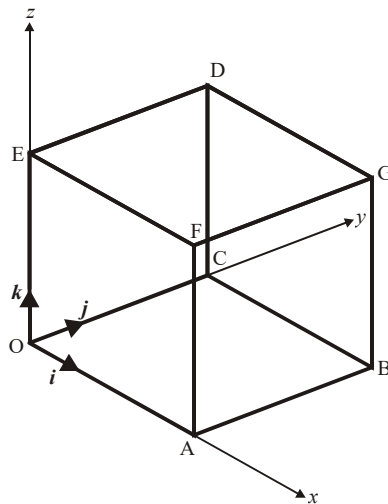
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3. [Maximum mark: 6]

The diagram shows a cube,  $OABCDEFG$  where the length of each edge is 5cm.

Express the following vectors in terms of  $i$ ,  $j$  and  $k$ .



$\vec{OG} =$
$\vec{BD} =$
$\vec{EB} =$

4. [Maximum mark: 20]

Let  $\mathbf{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ . Find

magnitude of $\mathbf{a}$	
magnitude of $\mathbf{b}$	
unit vector corresponding to $\mathbf{a}$	
unit vector corresponding to $\mathbf{b}$	
$\mathbf{a} + \mathbf{b}$	
$\mathbf{a} - \mathbf{b}$	
$2\mathbf{a}$	
$\mathbf{a} - 2\mathbf{b}$	
$ \mathbf{a} - 2\mathbf{b} $	
a vector $\mathbf{c}$ parallel to $\mathbf{a}$ of magnitude 39	

5. [Maximum mark: 16]

Let  $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ . Find

magnitude of $\mathbf{a}$	
magnitude of $\mathbf{b}$	
unit vector corresponding to $\mathbf{a}$	
unit vector corresponding to $\mathbf{b}$	
$\mathbf{a} + \mathbf{b}$	
$\mathbf{a} + 2\mathbf{b}$	
$ \mathbf{a} + 2\mathbf{b} $	
a vector $\mathbf{c}$ parallel to $\mathbf{a}$ of magnitude 6	

6. [Maximum mark: 4]

The vectors  $\mathbf{u}$ ,  $\mathbf{v}$  are given by  $\mathbf{u} = 3\mathbf{i} + 5\mathbf{j}$ ,  $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$ . Find scalars  $a$ ,  $b$  such that

$$a(\mathbf{u} + \mathbf{v}) = 8\mathbf{i} + (b - 2)\mathbf{j}.$$

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7. [Maximum mark: 5]

The vectors  $\vec{i}$ ,  $\vec{j}$  are unit vectors along the  $x$ -axis and  $y$ -axis respectively.

The vectors  $\vec{u} = -\vec{i} + 2\vec{j}$  and  $\vec{v} = 3\vec{i} + 5\vec{j}$  are given.

(a) Find  $\vec{u} + 2\vec{v}$  in terms of  $\vec{i}$  and  $\vec{j}$ . [2]

A vector  $\vec{w}$  has the same direction as  $\vec{u} + 2\vec{v}$ , and has a magnitude of 26.

(b) Find  $\vec{w}$  in terms of  $\vec{i}$  and  $\vec{j}$ . [3]

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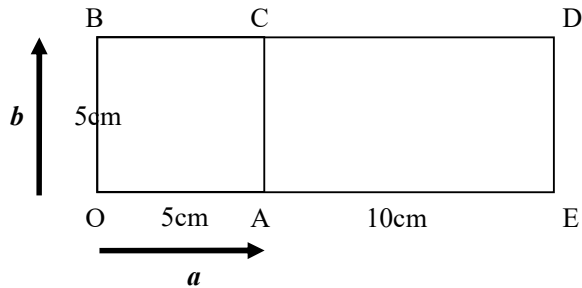
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9. [Maximum mark: 12]

In the following diagram OBCA is a square of side 5cm, while ACDE is a rectangle of length 10cm and width 5cm. Let  $\mathbf{a} = \overrightarrow{OA}$  and  $\mathbf{b} = \overrightarrow{OB}$



Use the geometric definition of the dot product to find the following

$\mathbf{a} \cdot \mathbf{b} =$
$\mathbf{a}^2 =$
$\mathbf{b}^2 =$
$\overrightarrow{OE} \cdot \overrightarrow{AC} =$
$\overrightarrow{OA} \cdot \overrightarrow{AE} =$
$\overrightarrow{OA} \cdot \overrightarrow{OC} =$

10. [Maximum mark: 6]

Let  $\mathbf{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ . Use the algebraic definition of the dot product to find

$\mathbf{a} \cdot \mathbf{b}$	
$\mathbf{a}^2$	
$\mathbf{b}^2$	
cosine of angle $\theta$ between $\mathbf{a}$ and $\mathbf{b}$	

11. [Maximum mark: 6]

Consider the vectors  $c = 3i + 4j$  and  $d = 5i - 12j$ .

- (a) Calculate the scalar product  $c \cdot d$ . [2]
- (b) Find the vector  $c + d$ . [1]
- (c) Find the value of  $|c| + |d|$ . [3]

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12. [Maximum mark: 6]

Find the cosine of the angle between the two vectors  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ .

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13. [Maximum mark: 4]

Find the size of the angle between the two vectors  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ . Give your answer to the nearest degree.

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16. [Maximum mark: 5]

Find the angle between the vectors  $\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$  and  $\mathbf{w} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ . Give your answer in radians.

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17. [Maximum mark: 6]

The position vectors of points P and Q are  $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix}$  respectively. The origin is at O

- (a) Find the angle  $\hat{P}OQ$ ; [4]
- (b) Find the area of the triangle OPQ. [2]

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18. [Maximum mark: 6]

A triangle has its vertices at  $A(-1, 3)$ ,  $B(3, 6)$  and  $C(-4, 4)$ .

(a) Show that  $\vec{AB} \cdot \vec{AC} = -9$ . [3]

(b) Show that, to three significant figures,  $\cos \hat{BAC} = -0.569$ . [3]

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19. [Maximum mark: 6]

A triangle has its vertices at  $A(-1, 3, 2)$ ,  $B(3, 6, 1)$  and  $C(-4, 4, 3)$ .

(a) Show that  $\vec{AB} \cdot \vec{AC} = -10$ . [3]

(b) Show that, to three significant figures,  $\cos \hat{BAC} = -0.591$ . [3]

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20. [Maximum mark: 6]

Consider the points  $A(5, 8)$ ,  $B(3, 5)$  and  $C(8, 6)$ .

(a) Find (i)  $\overrightarrow{AB}$ ; (ii)  $\overrightarrow{AC}$ . [3]

(b) (i) Find  $\overrightarrow{AB} \cdot \overrightarrow{AC}$ . (ii) Find the size of the angle between  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ . [3]

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21. [Maximum mark: 7]

Consider the points  $A(2,1,3)$ ,  $B(5,0,4)$  and  $C(4,7,3)$ .

(a) Find  $\overrightarrow{AB} \cdot \overrightarrow{AC}$ . [3]

(b) Find the size of the angle  $BAC$ . [1]

(c) Find the area of the triangle  $ABC$ . [3]

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22. [Maximum mark: 5]

The quadrilateral  $OABC$  has vertices with coordinates  $O(0,0)$   $A(5,1)$   $B(10,5)$  and  $C(2,7)$

- (a) Find the vectors  $\overrightarrow{OB}$  and  $\overrightarrow{AC}$ . [2]
- (b) Find the angle between the diagonals of the quadrilateral  $OABC$ . [3]

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23. [Maximum mark: 4]

The vectors  $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$  and  $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$  are perpendicular for two values of  $x$ .

- (a) Write down the quadratic equation which the two values of  $x$  must satisfy. [2]
- (b) Find the two values of  $x$ . [2]

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25. [Maximum mark: 6]

Consider the vectors  $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$  and  $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - p\mathbf{k}$ .

(a) Given that  $\mathbf{u}$  is perpendicular to  $\mathbf{v}$  find the value of  $p$ . [3]

(b) Given that  $q|\mathbf{u}|=14$ , find the value of  $q$ . [3]

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26. [Maximum mark: 7]

Let  $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$  and  $\mathbf{w} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ . The vector  $\mathbf{v} + p\mathbf{w}$  is perpendicular to  $\mathbf{w}$ .

Find the value of  $p$ .

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27. [Maximum mark: 6]

The vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are defined by  $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ , and  $\mathbf{c} = \begin{pmatrix} 2 \\ y \\ 3 \end{pmatrix}$ .

Given that  $\mathbf{c}$  is perpendicular to  $2\mathbf{a} - \mathbf{b}$ , find the value of  $y$ .

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28. [Maximum mark: 6]

Consider the vectors  $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$ ,  $\mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ \lambda \end{pmatrix}$ ,  $\mathbf{d} = \begin{pmatrix} \mu \\ -2 \\ 1 \end{pmatrix}$

Let  $\mathbf{s} = (\mathbf{a} \cdot \mathbf{b})\mathbf{c} + \mathbf{d}$ , where  $\mathbf{s}$  is perpendicular to  $\mathbf{a}$ .

Find an expression for  $\lambda$  in terms of  $\mu$ .

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29. [Maximum mark: 6]

The angle between the vector  $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$  and the vector  $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + m\mathbf{k}$  is  $30^\circ$ .

Find the values of  $m$ .

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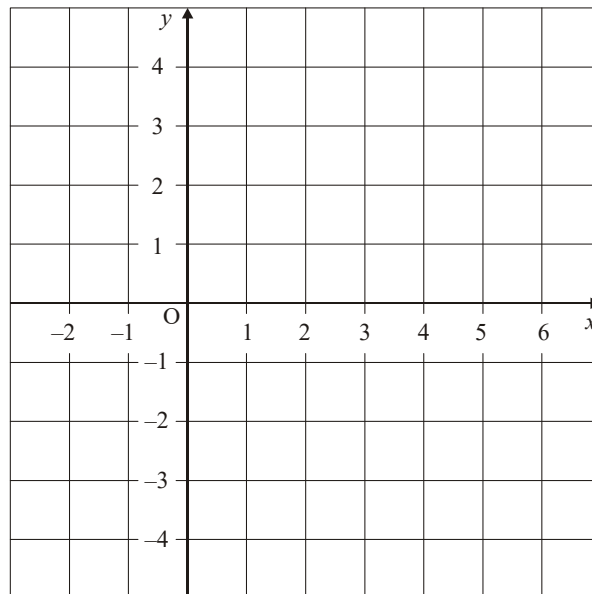
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30. [Maximum mark: 5]

The triangle ABC is defined by the following information

$$\vec{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad \vec{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad \vec{AB} \cdot \vec{BC} = 0, \quad \vec{AC}, \text{ is parallel to } \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

(a) On the grid below, draw an accurate diagram of triangle ABC. [3]



(b) Write down the vector  $\vec{OC}$ . [2]

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