

INTERNATIONAL BACCALAUREATE  
Mathematics: applications and interpretation

**MAI**

**EXERCISES [MAI 3.8]**  
**TRIGONOMETRIC FUNCTIONS**

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**A. Paper 1 questions (SHORT)**

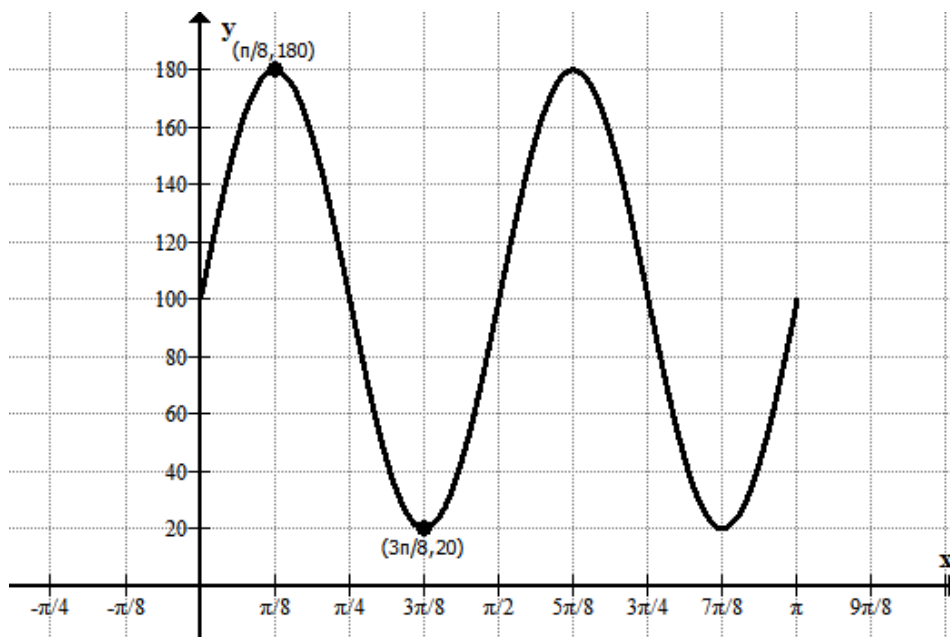
1. [Maximum mark: 14]

Complete the following table

Function	Amplitude	Period	Range
$f(x) = \sin x$	1	$2\pi$	$-1 \leq y \leq 1$
$f(x) = \cos x$			
$f(x) = \sin x + 1$			
$f(x) = \sin x - 1$			
$f(x) = 5 \sin x$			
$f(x) = -7 \sin x$			
$f(x) = \sin 4x$			
$f(x) = -\cos 4x$			
$f(x) = 3 \sin 4x$			
$f(x) = 3 \sin 4x + 10$			
$f(x) = 3 \sin 4x - 2$			
$f(x) = -5 \sin 3x$			
$f(x) = -5 \sin x + 10$			
$f(x) = -5 \sin x - 10$			
$f(x) = -5 \cos x - 10$			

2. [Maximum mark: 12]

Part of the graph of a trigonometric function  $f(x)$  is given below. There is a local maximum at  $(\pi/8, 180)$  and a local minimum at  $(3\pi/8, 20)$ .



- (a) Write down the values of  
 (i) the amplitude (ii) the central/principal axis (iii) the period [3]
- (b) Express the function in the form  $f(x) = A \sin Bx + C$  [3]
- (c) Complete the following table by expressing  $f(x)$  in three alternative forms. [6]

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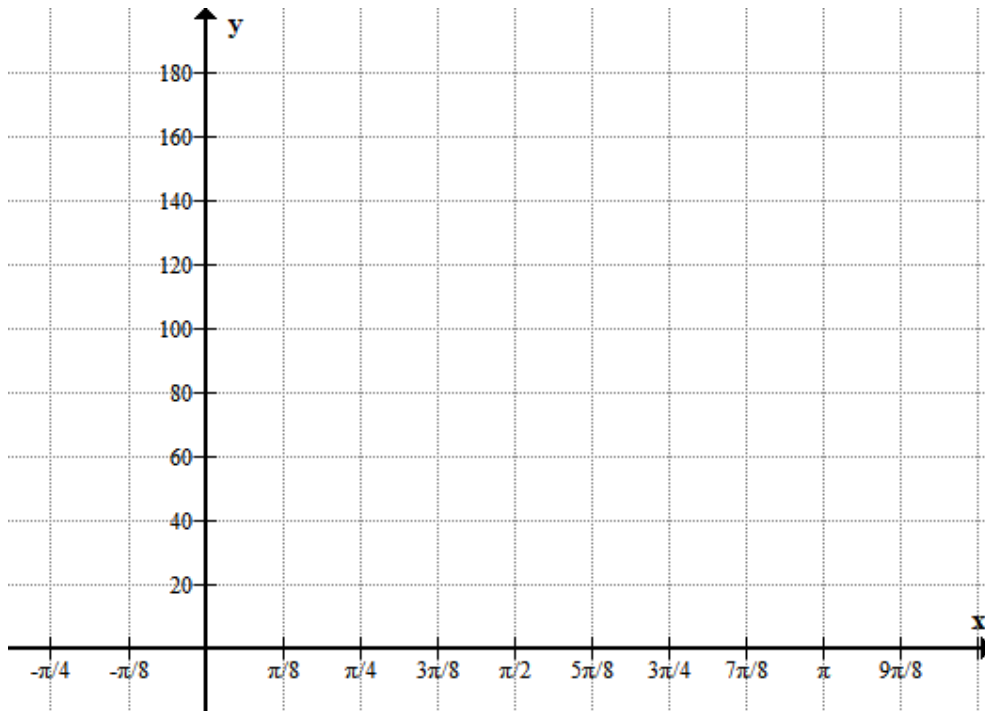
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$f(x) = -80 \sin[B(x - D)] + C$	
$f(x) = 80 \cos[B(x - D)] + C$	
$f(x) = -80 \cos[B(x - D)] + C$	

3. [Maximum mark: 4]

Sketch the graph of the function

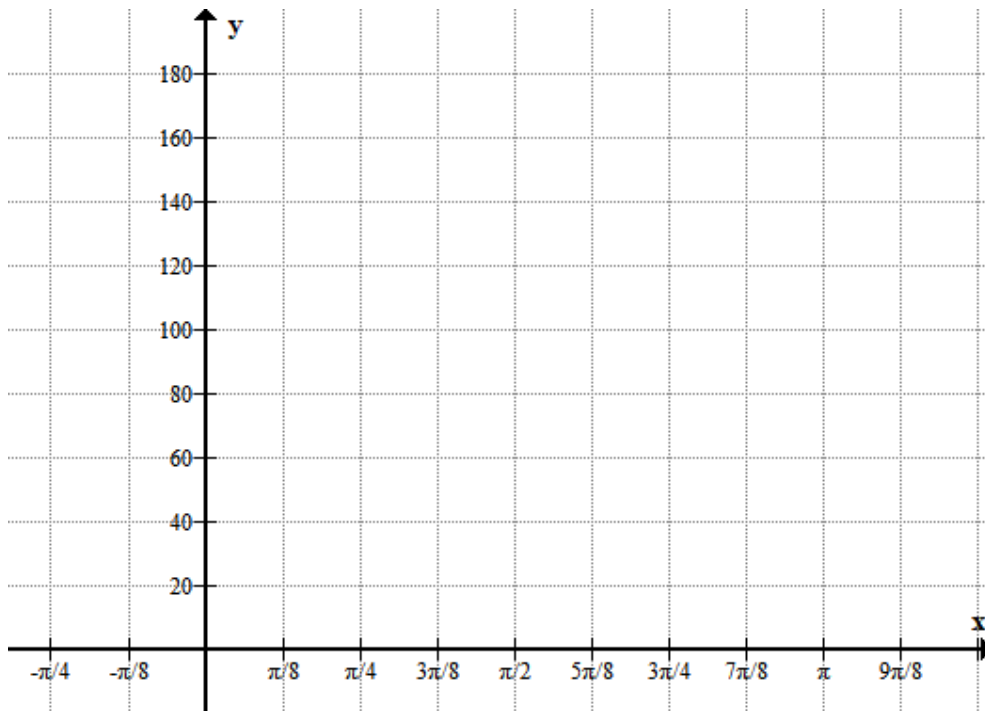
$$f(x) = 60 \sin 4x + 100, \quad 0 \leq x \leq \pi$$



4. [Maximum mark: 4]

Sketch the graph of the function

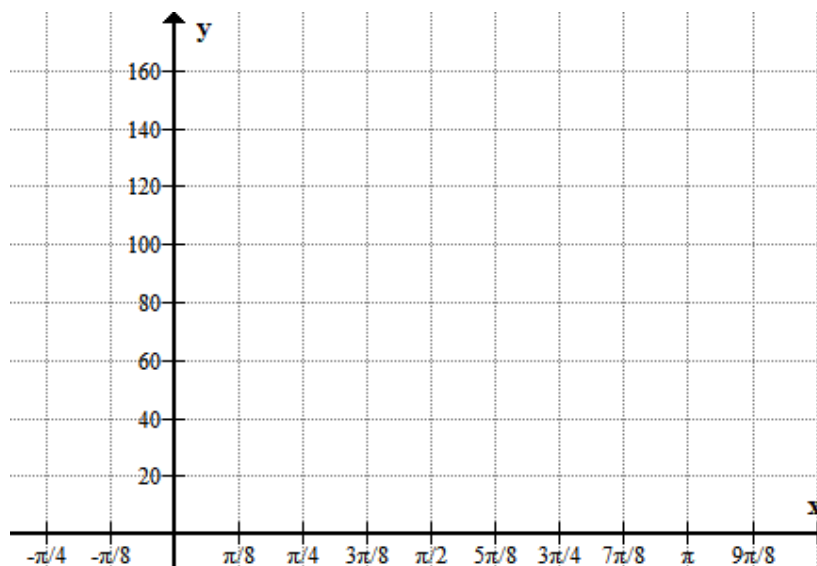
$$f(x) = 60 \cos 4x + 100, \quad 0 \leq x \leq \pi$$



5. [Maximum mark: 4]

Sketch the graph of the function

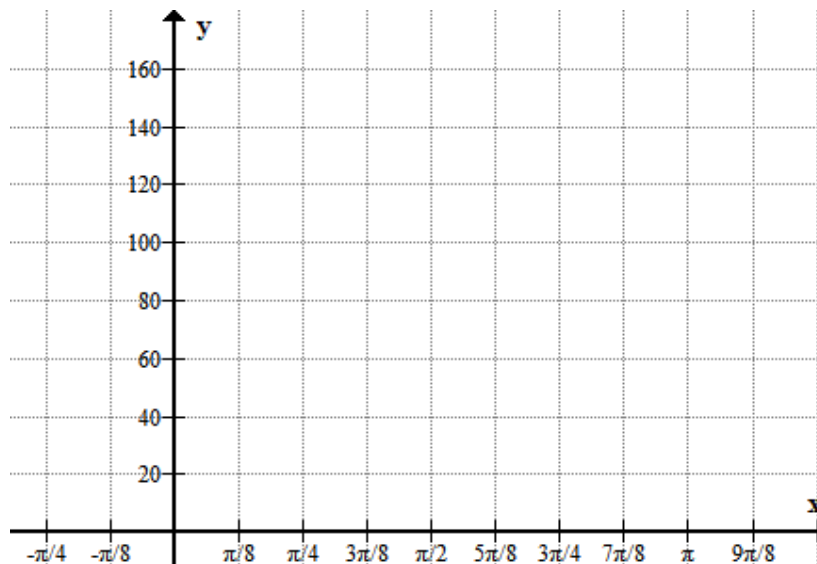
$$f(x) = -60\sin 4x + 80, \quad 0 \leq x \leq \pi$$



6. [Maximum mark: 8]

(a) Sketch the graph of the function

$$f(x) = -60\cos 4x + 80, \quad 0 \leq x \leq \pi$$



[4]

(b) Write down the possible values of  $k$  if the equation  $f(x) = k$  has

- (i) exactly two solution .....
- (ii) exactly three solutions .....
- (iii) exactly four solutions .....
- (iv) no solutions .....

[4]

7. [Maximum mark: 4]

Let  $f(x) = 4 \sin\left(3x + \frac{\pi}{2}\right)$ . For what values of  $k$  will the equation  $f(x) = k$  have no solutions?

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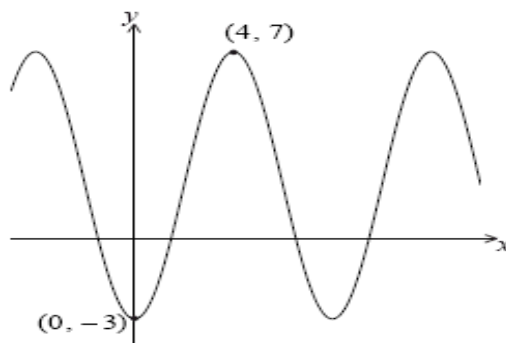
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8. [Maximum mark: 7]

The graph of  $y = p \cos qx + r$ , for  $-5 \leq x \leq 14$ , is shown below.



There is a minimum point at  $(0, -3)$  and a maximum point at  $(4, 7)$ .

- (a) Find the value of
- (i)  $p$ ;                      (ii)  $q$ ;                      (iii)  $r$ .                      [6]
- (b) The equation  $y = k$  has exactly **two** solutions. Write down the value of  $k$ .                      [1]

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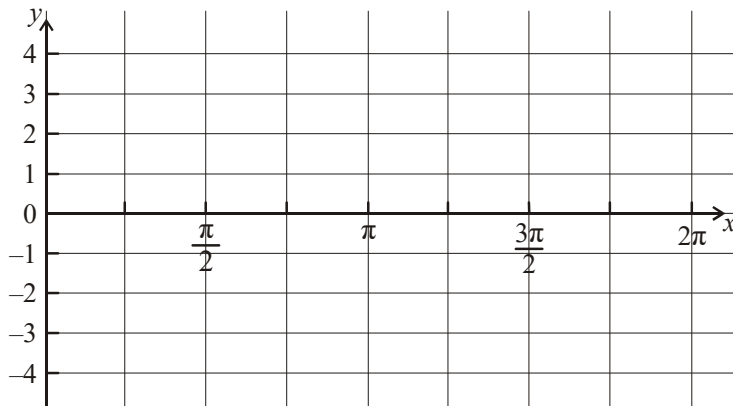
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9. [Maximum mark: 6]

Consider  $g(x) = 3 \sin 2x$ .

(a) Write down the period of  $g$ . [1]

(b) On the diagram below, sketch the curve of  $g$ , for  $0 \leq x \leq 2\pi$ .



(c) Write down the number of solutions to the equation  $g(x) = 2$ , for  $0 \leq x \leq 2\pi$ . [2]

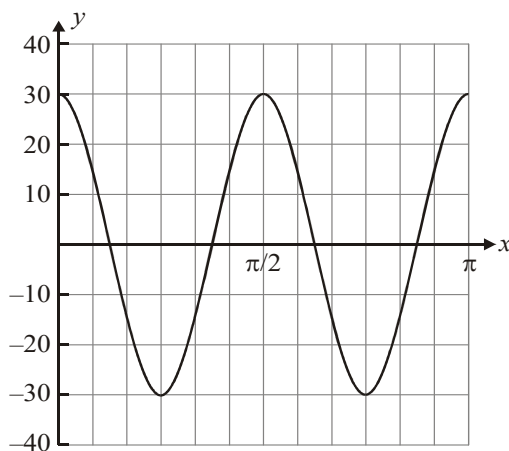
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10. [Maximum mark: 4]

The graph of a function of the form  $y = p \cos qx$  is given in the diagram below.



(a) Write down the value of  $p$ . [2]

(b) Calculate the value of  $q$ . [2]

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13. [Maximum mark: 4]

The depth,  $y$  metres, of sea water in a bay  $t$  hours after midnight may be represented by the function

$$y = a + b \cos\left(\frac{2\pi}{k}t\right) \quad \text{where } a, b \text{ and } k \text{ are constants.}$$

The water is at a maximum depth of 14.3 m at midnight and noon, and is at a minimum depth of 10.3 m at 06:00 and at 18:00.

Write down the value of (i)  $a$ ; (ii)  $b$ ; (iii)  $k$ .

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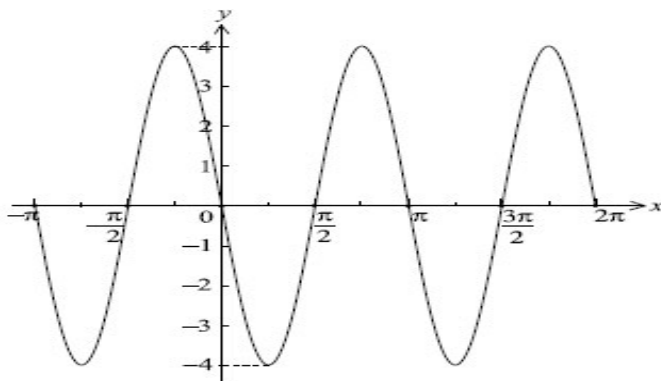
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14. [Maximum mark: 6]

Let  $f(x) = a \sin b(x - c)$ . Part of the graph of  $f$  is given below.



Given that  $a$ ,  $b$  and  $c$  are positive, find the value of  $a$ , of  $b$  and of  $c$ .

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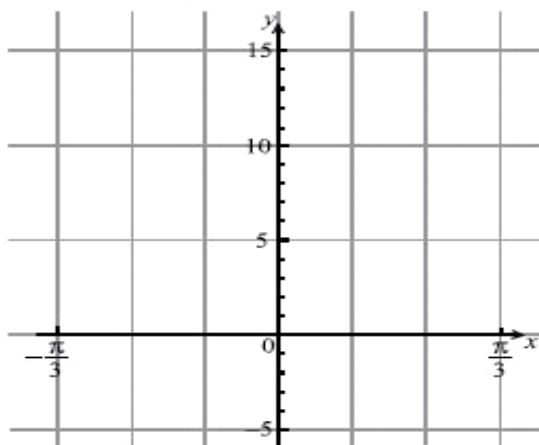
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15. [Maximum mark: 5]

Let  $f(x) = 4 \tan^2 x - 4 \sin x$ ,  $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$ .

(a) On the grid below, sketch the graph of  $y = f(x)$ .



[3]

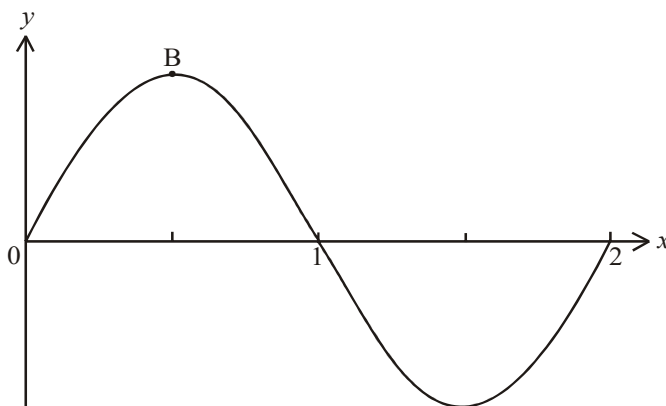
(b) Solve the equation  $f(x) = 1$ .

[2]

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16. [Maximum mark: 6]

Let  $f(x) = 6 \sin \pi x$  and  $g(x) = 6e^{-x} - 3$ , for  $0 \leq x \leq 2$ . The graph of  $f$  is shown on the diagram below. There is a maximum value at B  $(0.5, b)$ .



(a) Write down the value of  $b$ .

[1]

(b) On the same diagram, sketch the graph of  $g$ .

[3]

(c) Solve  $f(x) = g(x)$ ,  $0.5 \leq x \leq 1.5$ .

[2]

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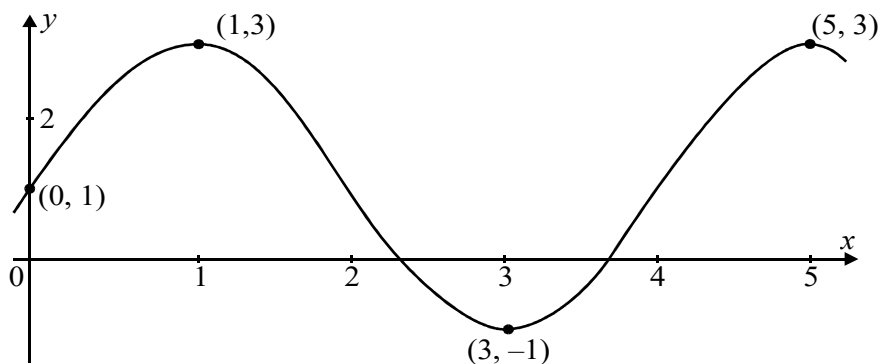


19. [Maximum mark: 18]

The diagram shows the graph of the function  $f$  given by

$$f(x) = A \sin\left(\frac{\pi}{2}x\right) + B, \text{ for } 0 \leq x \leq 5,$$

where  $A$  and  $B$  are constants, and  $x$  is measured in radians.



The graph includes the points  $(1, 3)$  and  $(5, 3)$ , which are maximum points of the graph.

- (a) Write down the values of  $f(1)$  and  $f(5)$ . [2]
- (b) Show that the period of  $f$  is 4. [2]

The point  $(3, -1)$  is a minimum point of the graph.

- (c) Show that  $A = 2$ , and find the value of  $B$ . [5]
- (d) Solve the equation  $f(x) = 2$  for  $0 \leq x \leq 5$ . [5]
- (e) Consider the equation  $f(x) = k$ , for  $0 \leq x \leq 5$ .

Write down the possible values of  $k$  if the equation has

- (i) exactly one solution
- (ii) exactly three solutions
- (iii) exactly two solutions
- (iv) no solutions [4]

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20. [Maximum mark: 13]

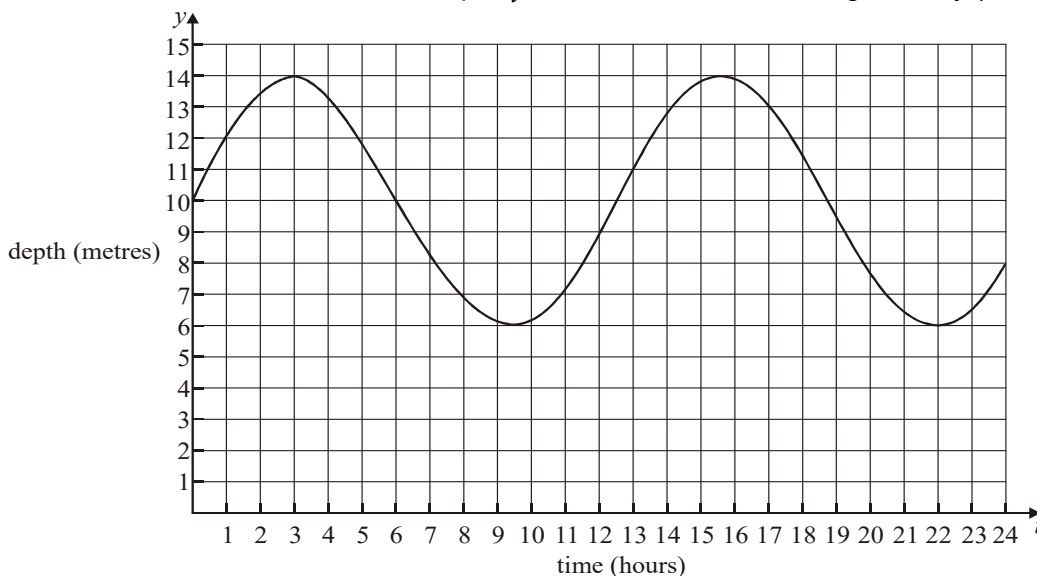
The depth  $y$  metres of water in a harbour is given by the equation

$$y = 10 + 4 \sin\left(\frac{t}{2}\right).$$

where  $t$  is the number of hours after midnight.

- (a) Calculate the depth of the water (i) when  $t = 2$ ; (ii) at 21:00. [3]

The sketch below shows the depth  $y$ , of water, at time  $t$ , during one day (24 hours).



- (b) (i) Write down the maximum depth of water in the harbour.  
 (ii) Calculate the value of  $t$  when the water is first at its maximum depth during the day. [3]

The harbour gates are closed when the depth of the water is less than seven metres. An alarm rings when the gates are opened or closed.

- (c) (i) How many times does the alarm sound during the day?  
 (ii) Find the value of  $t$  when the alarm sounds first.  
 (iii) Use the graph to find the length of time during the day when the harbour gates are closed. Give your answer in hours, to the nearest hour. [7]

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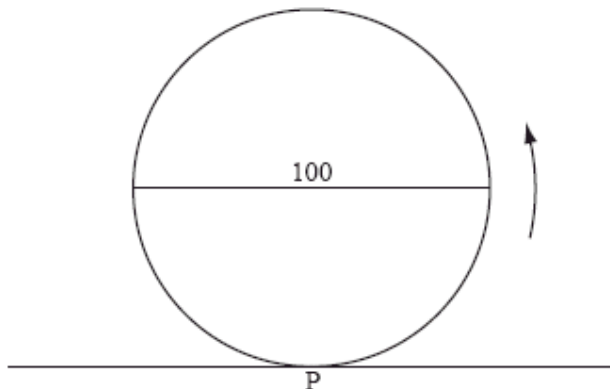






24. [Maximum mark: 14]

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction. One revolution takes 20 minutes.

- (a) Write down the height of P above ground level after  
 (i) 10 minutes;                      (ii) 15 minutes. [2]

Let  $h(t)$  metres be the height of P above ground level after  $t$  minutes. Some values of  $h(t)$  are given in the table below.

$t$	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

- (b) (i) Show that  $h(8) = 90.5$ .      (ii) Find  $h(21)$ . [4]  
 (c) **Sketch** the graph of  $h$ , for  $0 \leq t \leq 40$ . [3]  
 (d) Given that  $h$  can be expressed in the form  $h(t) = a \cos bt + c$ , find  $a$ ,  $b$  and  $c$ . [5]

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