INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 3.12-3.13]

EQUATIONS OF LINES

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 7]

Complete the following table

Passing through	Parallel to	Equation of line
A(3,5)	$\vec{b}=\begin{pmatrix}1\\7\end{pmatrix}$	$\vec{r}=\begin{pmatrix}3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\7\end{pmatrix}$
the origin	$\vec{b}=\begin{pmatrix}2\\3\end{pmatrix}$	
A(3,5)	x-axis	
A(3,5)	y-axis	
A(1,3,5)	$\vec{b}=\begin{pmatrix}1\\7\\2\end{pmatrix}$	$\vec{r}=\begin{pmatrix}1\\3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\7\\2\end{pmatrix}$
the origin	$\vec{b}=\begin{pmatrix}1\\4\\3\end{pmatrix}$	
A(1,3,5)	x-axis	
A(1,3,5)	y-axis	
A(1,3,5)	z-axis	

2. [Maximum mark: 8]

Complete the following table

Passing through	Equation of line
A(3,5) and *B*(4,12)	$\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$
A(1,-4) and *B*(7,6)	
the origin and *B*(7,6)	
A(1,3,5) and *B*(2,10,7)	$\vec{r} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$
A(1,3,5) and *B*(0,5,3)	
the origin and *B*(2,10,7)	

3. [Maximum mark: 1]

Express the following 3D lines in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$

(i) x-axis (ii) y-axis (iii) z-axis

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4. [Maximum mark: 6]

Determine whether the line $\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ passes through

(i) A(5,19) (ii) B(5,20) (iii) the origin

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5. [Maximum mark: 4]

Find a vector equation of the line passing through (–1, 4) and (3, –1). Give your answer in the form $\boldsymbol{r} = \boldsymbol{p} + t\boldsymbol{d}$, where $t \in \mathbb{R}$.

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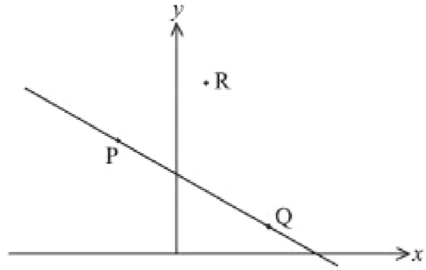
6. [Maximum mark: 4]

The points $P(-2, 4)$, $Q(3, 1)$ and $R(1, 6)$ are shown in the diagram below.



(a) Find the vector $\overrightarrow{PQ}$. [2]

(b) Find a vector equation for the line through R parallel to the line (PQ). [2]

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7. [Maximum mark: 4]

The line *L* passes through the origin and is parallel to the vector $2\boldsymbol{i} + 3\boldsymbol{j}$.

(a) Write down a vector equation for *L*.

(b) Find the Cartesian equation of the line in the form $y=mx$

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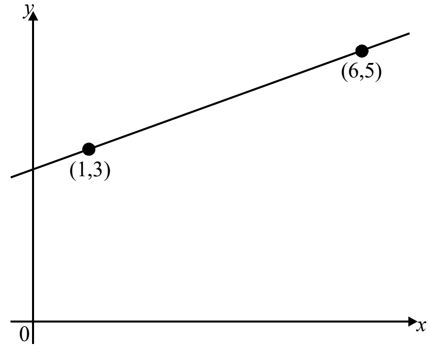
8. [Maximum mark: 4]

The diagram below shows a line passing through the points (1,3) and (6,5).



Find a vector equation for the line in the form $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + t\begin{pmatrix} c \\ d \end{pmatrix}$, where $t \in$ R

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9. [Maximum mark: 6]

A vector equation of a line is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $t \in$R.

Find the equation of this line in the form $ax + by = c$, where a, b, and $c \in$Z.

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10. [Maximum mark: 4]

A line passes through the point (4,–1). Its direction is perpendicular to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Find the equation of the line in the form $ax + by = p$, where a, b and p are integers.

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11. [Maximum mark: 5]

The line *L* passes through the points A (3, 2, 1) and B (1, 5, 3).

(a) Find the vector $\overrightarrow{AB}$. [3]

(b) Write down a vector equation of the line *L* in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. [2]

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12. [Maximum mark: 4]

A vector equation for the line L is $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Which of the following are also vector equations for the same line L?

A. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

B. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

C. $\boldsymbol{r} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

D. $\boldsymbol{r} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

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13. [Maximum mark: 1]

Given the points $A(1,1,1)$ $B(2,2,2)$ $C(5,5,5)$ $D(4,5,6)$

(a) Find the vectors AB and BC and hence explain why *A, B* and *C* are collinear

(b) Find the vector equation of the line *AD*

(c) Find the cosine of the angle *BAD*

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14. [Maximum mark: 5]

Calculate the acute angle between the lines with equations

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

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15. [Maximum mark: 5]

Two lines L_1 and L_2 have these vector equations.

$L_1 : \boldsymbol{r} = 2\boldsymbol{i} + 3\boldsymbol{j} + t(\boldsymbol{i} - 3\boldsymbol{j})$ $\qquad$ $L_2 : \boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + s(\boldsymbol{i} - \boldsymbol{j})$

The angle between L_1 and L_2 is θ. Find the cosine of the angle θ.

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16. [Maximum mark: 6]

The lines L_1 and L_2 have parametric equations

$L_1 : x = 1 + 2\lambda,\ y = 1 + 3\lambda,\ z = 1 - \lambda$ $\qquad$ $L_2 : x = 2 - \mu,\ y = 3 + 4\mu,\ z = 4 + 2\mu$

Find the angle between L_1 and L_2.

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17. [Maximum mark: 6]

The vector equations of two lines are given below.

$$\boldsymbol{r}_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + s\begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \boldsymbol{r}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The lines intersect at the point P. Find the position vector of P.

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18. [Maximum mark: 6]

The lines l_1 and l_2 have equations

$$\boldsymbol{r}_1 = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} \text{ and } \boldsymbol{r}_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$$

respectively, where λ and μ are parameters.

(a) Show that l_1 passes through the point (2, –7 , 4).

(b) Determine whether the lines l_1 and l_2 intersect.

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19. [Maximum mark: 10]

Consider the lines L_1: $\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ and L_2: $\vec{r} = \begin{pmatrix} -2 \\ 20 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -1 \end{pmatrix}$

(a) Show that the lines are perpendicular. [2]

(b) Find the point of intersection of the two lines. [3]

(c) Find the Cartesian equations of the two lines in the form $ax + by = c$ [4]

(d) Solve the simultaneous equations in (c) [1]

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20. [Maximum mark: 7]

Consider the lines L_1: $\vec{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ and L_2: $\vec{r} = \begin{pmatrix} 2 \\ 9 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$

(a) Show that the two lines intersect. [4]

(b) Find the point of intersection. [1]

(c) Find the angle between the two lines. [2]

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21. [Maximum mark: 6]

The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

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22. [Maximum mark: 6]

The vector equations of the lines L_1 and L_2 are given by

$$L_1\text{: } \boldsymbol{r} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} + \lambda(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}); \qquad L_2\text{: } \boldsymbol{r} = \boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k} + \mu(2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}).$$

The two lines intersect at the point P. Find the position vector of P.

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23. [Maximum mark: 6]

Consider the four points A(1, 4, –1), B(2, 5, –2), C(5, 6, 3) and D(8, 8, 4). Find the point of intersection of the lines (AB) and (CD).

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24. [Maximum mark: 5]

Point A(3, 0, –2) lies on the line $\boldsymbol{r} = 3\boldsymbol{i} - 2\boldsymbol{k} + \lambda(2\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$, where λ is a real parameter. Find the coordinates of **one** point which is 6 units from A, and on the line.

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25. [Maximum mark: 6]

The line *L* passes through A (0, 3) and B (1, 0). The origin is at O. The point R$(x, 3-3x)$ is on *L*, and (OR) is perpendicular to *L*.

(a) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{OR}$. [3]

(b) Use the scalar product to find the coordinates of R. [3]

26. [Maximum mark: 7]

Consider the points A (1, 3, –17) and B (6, – 7, 8) which lie on the line *l*.

(a) Find an equation of line *l*, giving the answer in parametric form. [4]

(b) The point *P* is on *l* such that $\overrightarrow{OP}$ is perpendicular to *l*. Find the coordinates of P. [3]

27. [Maximum mark: 14]

The line *L* has vector equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.

The coordinates of a point P on *L* are of the form $P(2+2\lambda, \lambda, 3+2\lambda)$

(a) Determine whether the following points lie on *L*: (i) $A(8,3,8)$ (ii) $B(8,3,9)$. [2]

(b) Find two points on *L*, 6 units far from B. [4]

(c) Find two points on *L*, $\sqrt{89}$ units far from the origin. [4]

(d) Find two points on *L*, $\sqrt{54}$ units far from $C(1,0,2)$. [4]

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28. [Maximum mark: 7]

The line *L* has vector equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$. The point $D\,(0,1,0)$ does not lie on *L*.

The coordinates of a point P on *L* are of the form $P\,(2+2\lambda,\ \lambda,\ 3+2\lambda)$.

(a) Find the point on *L* which is closest to the point D. [4]

(b) Find the distance between the line *L* and the point D. [1]

(c) The point D' is the reflection of D in the line *L*. Find the coordinates of D'. [2]

29. [Maximum mark: 6+6]

The line L is given by the parametric equations $x = 1 - \lambda$, $y = 2 - 3\lambda$, $z = 2$.

Find the coordinates of the point on L that is nearest to the origin.

Use two different methods.

METHOD A (use the fact that the vector OP is perpendicular to the line). [6]

METHOD B (use the formula of the distance d and then differentiation). [6]

30. [Maximum mark: 9]

The line *L* has vector equation

$$\vec{r} = 5\vec{i} + 9\vec{j} + 6\vec{k} + t(\vec{i} + 2\vec{j} + 2\vec{k}) \quad \text{where } t \text{ is a scalar.}$$

(a) Find the coordinates of the point on line *L* which is nearest to point A(0, 2, 2). [5]

(b) Calculate the shortest distance from the point *A*(0, 2, 2) to the line. [2]

(c) The point A is reflected in line *L*. Find the coordinates of the image A´. [2]

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31. [Maximum mark: 8]

Two lines L_1 and L_2 are given by $\boldsymbol{r}_1 = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} + s\begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t\begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$.

(a) Let θ be the acute angle between L_1 and L_2. Show that $\cos\theta = \frac{52}{140}$. [3]

(b) (i) P is the point on L_1 when $s = 1$. Find the position vector of P.

(ii) Show that P is also on L_2. [3]

(c) A third line L_3 has direction vector $\begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$. If L_1, L_3 are parallel, find the value of x. [2]

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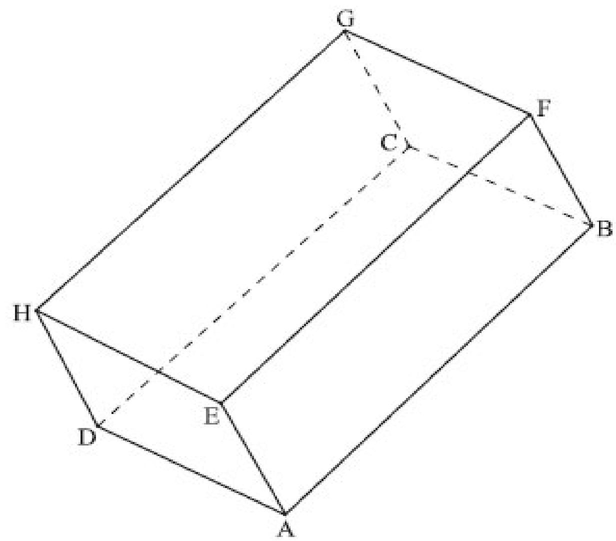
B. Paper 2 questions (LONG)

32. [Maximum mark: 18]

The following diagram shows a solid figure ABCDEFGH. Each of the six faces is a parallelogram.



The coordinates of A and B are A (7, –3, –5), B(17, 2, 5).

(a) Find (i) $\overrightarrow{AB}$; (ii) $\left|\overrightarrow{AB}\right|$. [4]

The following information is given. $\overrightarrow{AD} = \begin{pmatrix} -6 \\ 6 \\ 3 \end{pmatrix}$, $\left|\overrightarrow{AD}\right|$ = 9, $\overrightarrow{AE} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix}$, $\left|\overrightarrow{AE}\right|$ = 6

(b) (i) Calculate (a) $\overrightarrow{AD} \bullet \overrightarrow{AE}$ (b) $\overrightarrow{AB} \bullet \overrightarrow{AD}$ (c) $\overrightarrow{AB} \bullet \overrightarrow{AE}$

(ii) Hence, write down the size of the angle between any two intersecting edges [5]

(c) Calculate the volume of the solid ABCDEFGH. [2]

(d) The coordinates of G are (9, 14, 12). Find the coordinates of H. [3]

(e) The lines (AG), (HB) intersect at point P. Find the acute angle at P. [4]

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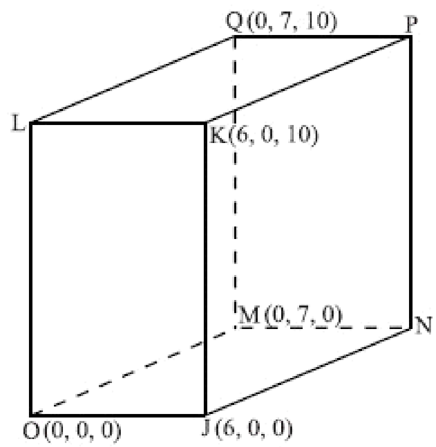
33. [Maximum mark: 13]

The diagram below shows a cuboid (rectangular solid) OJKLMNPQ.

The vertex O is (0, 0, 0), J is (6, 0, 0), K is (6, 0, 10), M is (0, 7, 0) and Q is (0, 7, 10).



(a) Find in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ the equations of lines: (i) (JQ) (ii) (MK) [6]

(b) Find the acute angle between (JQ) and (MK). [2]

(c) The lines (JQ) and (MK) intersect at D. Find the position vector of D. [5]

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34. [Maximum mark: 13]

Points A, B, and C have position vectors $4\boldsymbol{i} + 2\boldsymbol{j}$, $\boldsymbol{i} - 3\boldsymbol{j}$ and $-5\boldsymbol{i} - 5\boldsymbol{j}$. Let D be a point on the x-axis such that ABCD forms a parallelogram.

(a) (i) Find $\overrightarrow{\mathrm{BC}}$. (ii) Find the position vector of D. [4]

(b) Find the angle between $\overrightarrow{\mathrm{BD}}$ and $\overrightarrow{\mathrm{AC}}$. [4]

The line L_1 passes through A and is parallel to $\boldsymbol{i} + 4\boldsymbol{j}$.

The line L_2 passes through B and is parallel to $2\boldsymbol{i} + 7\boldsymbol{j}$.

A vector equation of L_1 is $\boldsymbol{r} = (4\boldsymbol{i} + 2\boldsymbol{j}) + s(\boldsymbol{i} + 4\boldsymbol{j})$.

(c) Write down a vector equation of L_2 in the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{q}$. [1]

(d) The lines L_1 and L_2 intersect at the point P. Find the position vector of P. [4]

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35. [Maximum mark: 14]

The point O has coordinates (0, 0, 0), point A has coordinates (1, –2, 3) and point B has coordinates (–3, 4, 2).

(a) (i) Find $\overrightarrow{AB}$ (ii) Find $B\hat{A}O$. [5]

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1. [2]

(c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

(i) Find a vector equation for L_2, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Point C $(k, -k, 5)$ is on L_2. Find the coordinates of C. [5]

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C. [2]

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36. [Maximum mark: 15]

Three of the coordinates of the parallelogram STUV are S(–2, –2), T(7, 7), U(5, 15).

(a) Find the vector $\overrightarrow{ST}$ and hence the coordinates of V. [4]

(b) Find a vector equation of the line (UV) in the form $\boldsymbol{r} = \boldsymbol{p} + t\mathbf{d}$ where $t \in \mathbb{R}$. [2]

(c) Show that the point E with position vector $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$ is on the line (UV), and find the value of t for this point. [2]

The point W has position vector $\begin{pmatrix} a \\ 17 \end{pmatrix}$, $a \in \mathbb{R}$.

(d) (i) If $|\overrightarrow{EW}| = 2\sqrt{13}$, show that one value of a is -3 and find the other possible value of a.

(ii) For $a = -3$, calculate the angle between $\overrightarrow{EW}$ and $\overrightarrow{ET}$. [7]

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37. [Maximum mark: 16]

Points P and Q have position vectors $-5\boldsymbol{i}+11\boldsymbol{j}-8\boldsymbol{k}$ and $-4\boldsymbol{i}+9\boldsymbol{j}-5\boldsymbol{k}$ respectively, and both lie on a line L_1.

(a) (i) Find $\overrightarrow{\mathrm{PQ}}$.

(ii) Hence show that the equation of L_1 can be written as

$$\boldsymbol{r} = (-5+s)\,\boldsymbol{i} + (11-2s)\,\boldsymbol{j} + (-8+3s)\,\boldsymbol{k}.$$ [4]

The point R $(2, y_1, z_1)$ also lies on L_1.

(b) Find the value of y_1 and of z_1. [3]

The line L_2 has equation $\boldsymbol{r} = 2\boldsymbol{i}+9\boldsymbol{j}+13\boldsymbol{k}+t\,(\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k})$.

(c) The lines L_1 and L_2 intersect at a point T. Find the position vector of T. [5]

(d) Calculate the angle between the lines L_1 and L_2. [4]

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38. [Maximum mark: 19]

The position vector of point A is $2\boldsymbol{i}+3\boldsymbol{j}+\boldsymbol{k}$ and the position vector of point B is $4\boldsymbol{i}-5\boldsymbol{j}+21\boldsymbol{k}$

(a) (i) Show that $\overrightarrow{AB} = 2\boldsymbol{i}-8\boldsymbol{j}+20\boldsymbol{k}$.

(ii) Find the unit vector $\boldsymbol{u}$ in the direction of $\overrightarrow{AB}$.

(iii) Show that $\boldsymbol{u}$ is perpendicular to $\overrightarrow{OA}$. [6]

Let S be the midpoint of [AB]. The line L_1 passes through S and is parallel to $\overrightarrow{OA}$.

(b) (i) Find the position vector of S.

(ii) Write down the equation of L_1. [4]

The line L_2 has equation $\boldsymbol{r} = (5\boldsymbol{i}+10\boldsymbol{j}+10\boldsymbol{k}) + s(-2\boldsymbol{i}+5\boldsymbol{j}-3\boldsymbol{k})$.

(c) Explain why L_1 and L_2 are not parallel. [2]

(d) The lines L_1 and L_2 intersect at the point P. Find the position vector of P. [7]

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39. [Maximum mark: 22]

The lines l_1 and l_2 have vector equations

$$\boldsymbol{r_1} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \text{ and } \boldsymbol{r_2} = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

respectively, where λ and μ are parameters.

(a) Find the acute angle between l_1 and l_2. [2]

(b) Find the acute angle between l_1 and z-*axis.* [2]

(c) Find the coordinates of the point of intersection of l_1 and l_2. [4]

(d) Find the reflection of point A(1,4,3) in line l_2. [5]

(e) Find the reflection of line l_1 in line l_2. [2]

Let $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$ be the direction vectors of the two lines.

(f) (i) Dhow that $|\boldsymbol{d_1}| = |\boldsymbol{d_2}|$.

(ii) Write down the vectors $\boldsymbol{d_1} + \boldsymbol{d_2}$ and $\boldsymbol{d_1} - \boldsymbol{d_2}$.

(i) **Hence** find the equations of the two bisector lines between l_1 and l_2. [7]

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40. [Maximum mark: 17]

The triangle ABC has vertices at the points A(–1, 2, 3), B(–l, 3, 5) and C(0, –1, 1).

(a) Find the size of the angle θ between the vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. [4]

(b) Hence, or otherwise, find the area of triangle ABC. [2]

Let l_1 be the line parallel to $\overrightarrow{\mathrm{AB}}$ which passes through D(2, –1, 0) and l_2 be the line parallel to $\overrightarrow{\mathrm{AC}}$ which passes through E(–1, 1, 1).

(c) (i) Find the equations of the lines l_1 and l_2.

(ii) Hence show that l_1 and l_2 do not intersect. [5]

(d) Find the shortest distance between l_1 and l_2. [6]

41. [Maximum mark: 14]

Consider the points A(1, −1, 4), B(2, − 2, 5) and O(0, 0, 0).

(a) Calculate the cosine of the angle between $\overrightarrow{OA}$ and $\overrightarrow{AB}$. [5]

(b) Find a vector equation of the line L_1 which passes through A and B. [2]

The line L_2 has equation $\boldsymbol{r} = 2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$, where $t \in \mathbb{R}$.

(c) Show that the lines L_1 and L_2 intersect and find the coordinates of their point of intersection. [7]

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