INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.9]

TRANSFORMATIONS

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 9]

The point A(1, 0.5) lies on the curve $y = f(x)$. Write down the coordinates of the corresponding point under the following transformations

$y = f(x) + 5$	(1, 5.5)	$y = f(x+5)$	
$y = f(x) - 5$		$y = f(x-5)$	
$y = 5f(x)$		$y = f(5x)$	
$y = f(x)/5$		$y = f(x/5)$	
$y = -f(x)$		$y = f(-x)$	

2. [Maximum mark: 12]

The point A(-1, 3) lies on the curve $y = f(x)$.

(a) Write down the coordinates of the corresponding point under the following transformations [9]

$y = f(x) + 3$	(-1, 6)	$y = f(x+3)$	
$y = f(x) - 3$		$y = f(x-3)$	
$y = 3f(x)$		$y = f(3x)$	
$y = f(x)/3$		$y = f(x/3)$	
$y = -f(x)$		$y = f(-x)$	

(b) Find the coordinates of the corresponding point under $y = 2f(x\text{-}3)+4$ [3]

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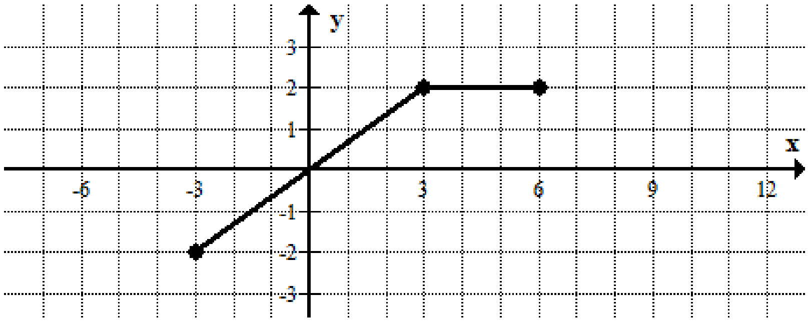
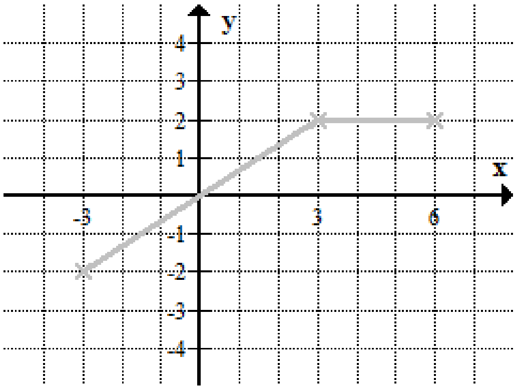
3. [Maximum mark: 10]

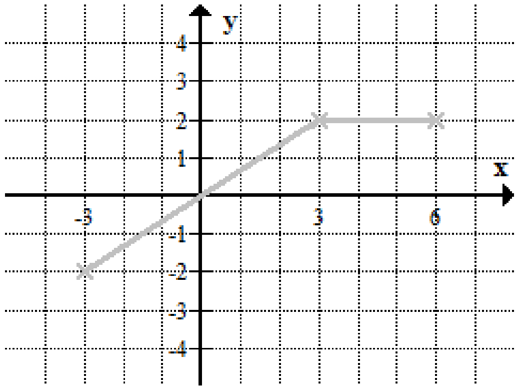
The graph of $y = f(x)$ is shown in the diagram.



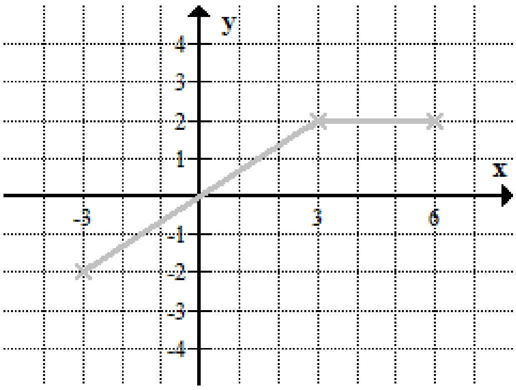
On each of the following diagrams draw the required graph.

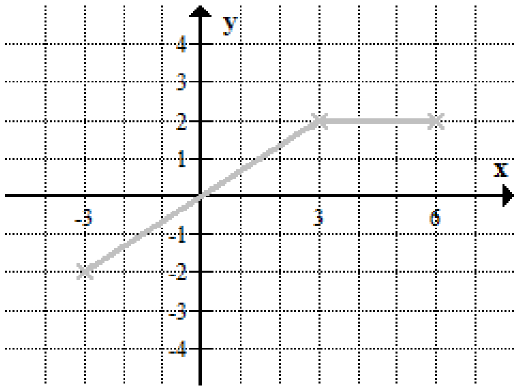
(a) $y = f(x)+1$ (b) $y = f(x)-2$ [1+1]



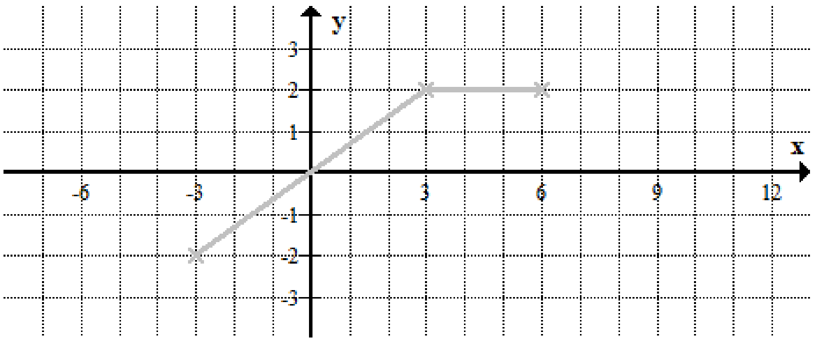


(c) $y = 2f(x)$ (d) $y = \frac{1}{2}f(x)$ [1+1]

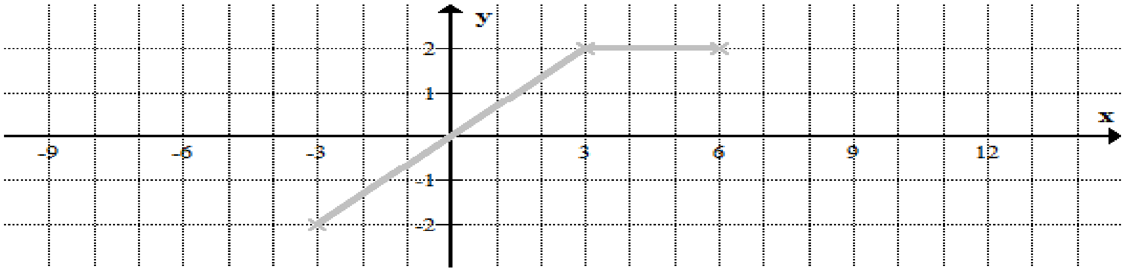




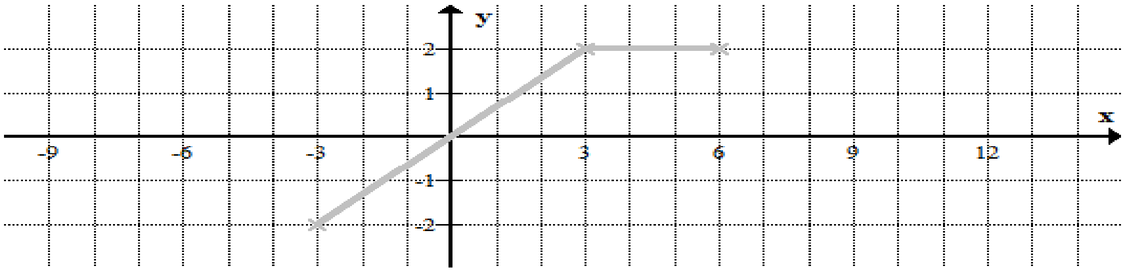
(e) $y = -f(x)$ [1]



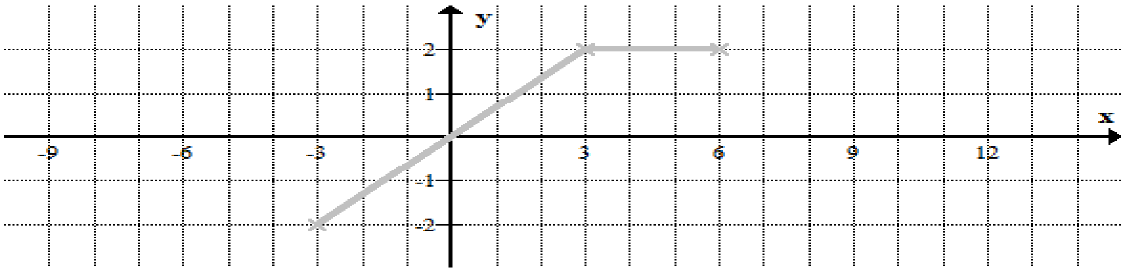
(f) $y = f(x+3)$ [1]



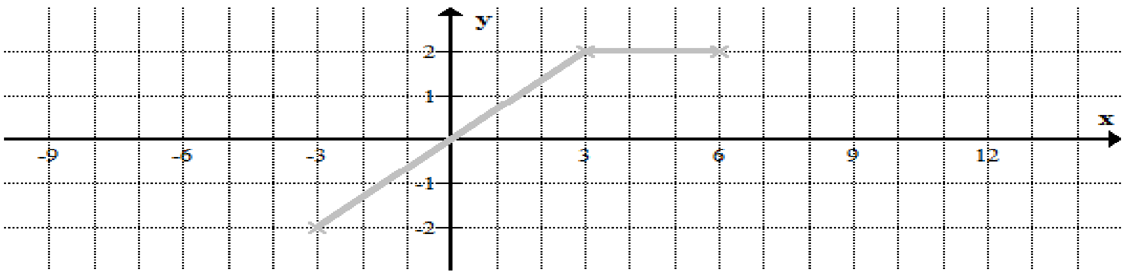
(g) $y = f(x-3)$ [1]



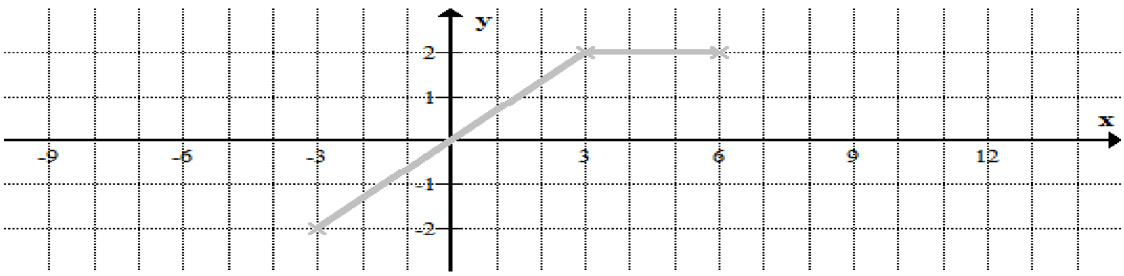
(h) $y = f(3x)$ [1]



(i) $y = f\left(\frac{x}{2}\right)$ [1]



(j) $y = f(-x)$ [1]



4. [Maximum mark: 11]

The sequence of transformations from $y = f(x)$ to $y = 2f(3x) + 4$ is given below

$f(x)$	original
$2f(x)$	vertical stretch with s.f. 2
$2f(3x)$	horizontal stretch with s.f. 1/3 (i.e. shrink))
$2f(3x)+4$	vertical translation 4 units up

Describe similarly the sequence of transformations, in a correct order, for the following functions

(a) $-f(x-2)+5$ [3]

$f(x)$	original

(b) $-3f(x+2)-1$ [4]

$f(x)$	original

(c) $f(2x-10)$ [2]

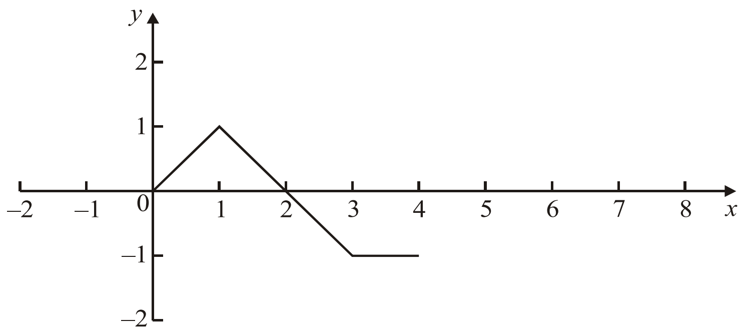
$f(x)$	original

(d) $f(2(x-5))$ [2]

$f(x)$	original

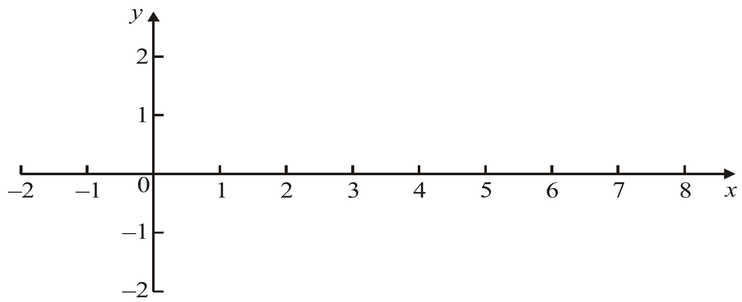
5. [Maximum mark: 6]

The graph of $y = f(x)$ is shown in the diagram.

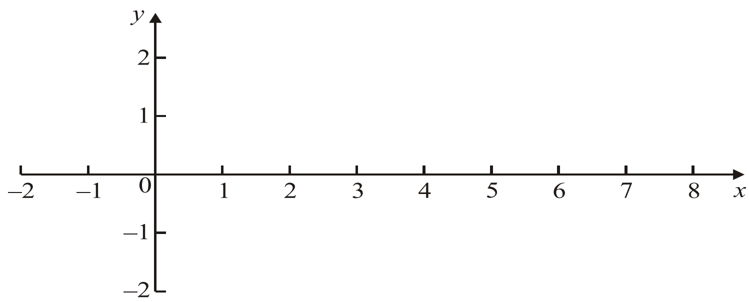


(a) On each of the following diagrams draw the required graph,

(i) $y = 2f(x)$;



(ii) $y = f(x-3)$.



[4]

(b) The point A (3, –1) is on the graph of *f*. The point A' is the corresponding point on the graph of $y = -f(x) + 1$.

Find the coordinates of A'. [2]

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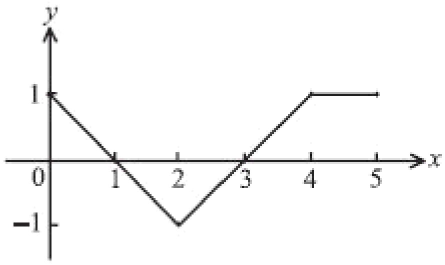
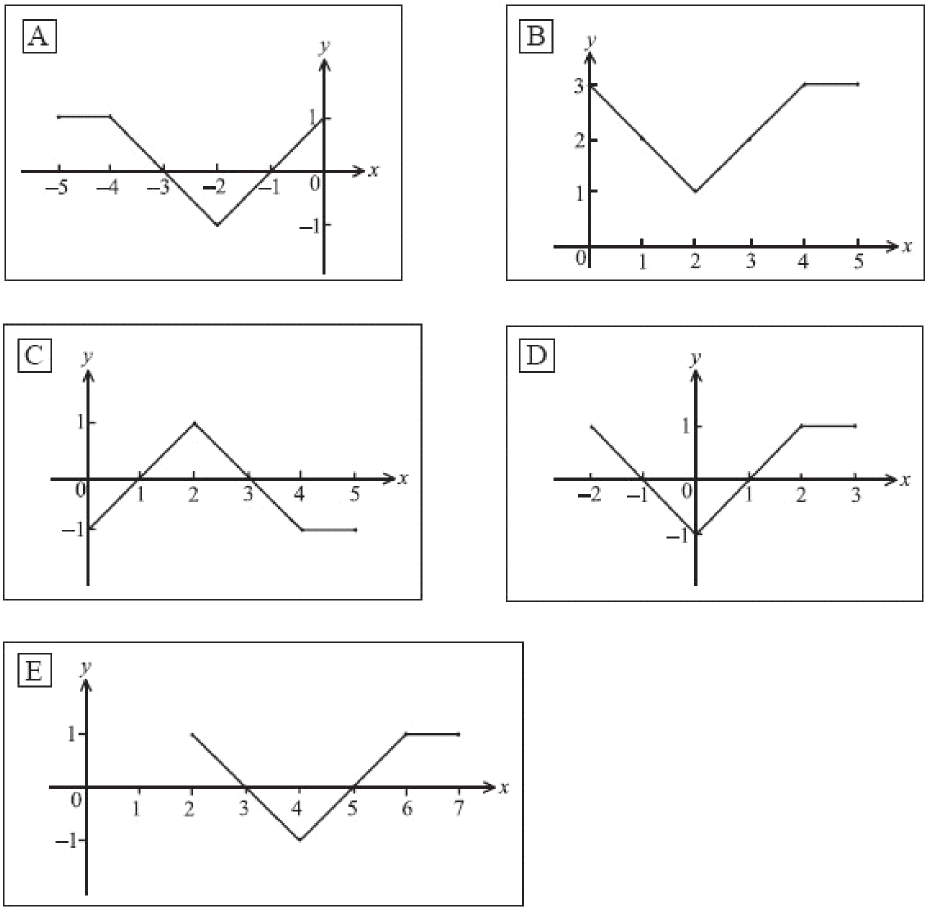
6. [Maximum mark: 5]

The following diagram shows part of the graph of $f(x)$



Consider the five graphs in the diagrams labelled A, B, C, D, E below.



(a) Which diagram is the graph of

(i) $f(x+2)$? (ii) $-f(x)$? (iii) $f(-x)$ [3]

(b) Write down the expression for each of the two remaining graphs [2]

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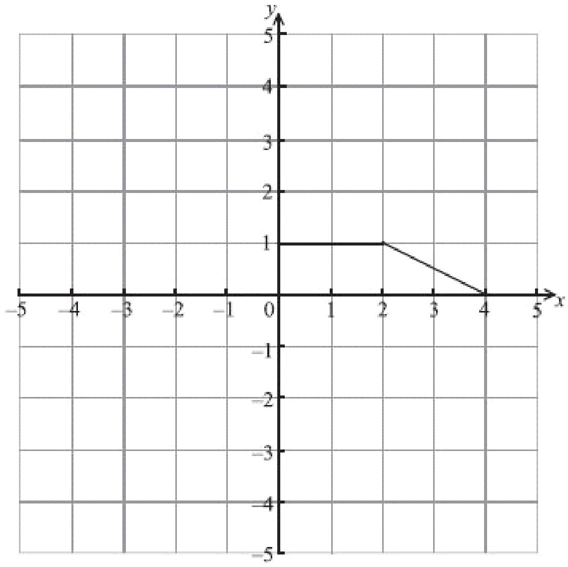
7. [Maximum mark: 10]

The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.



(a) Write down the value of (i) $f(1)$ (ii) $f(3)$. [2]

(b) On the diagrams below, draw the graphs of $y = 3f(-x)$ and $y = f(2x)$. [4]

(c) Write down the domain and the range of $y = 3f(-x)$ and $y = f(2x)$. [4]

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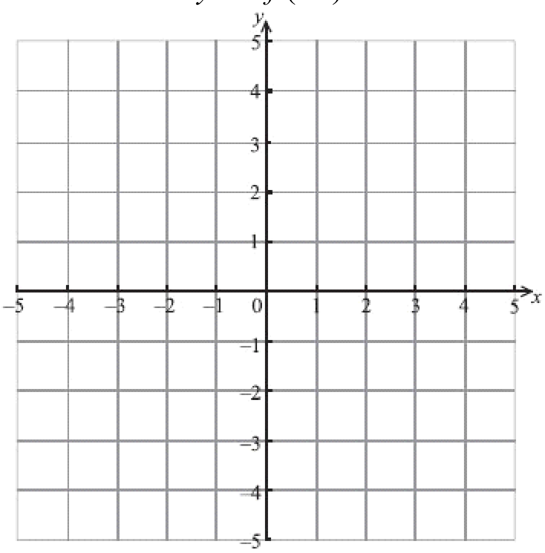
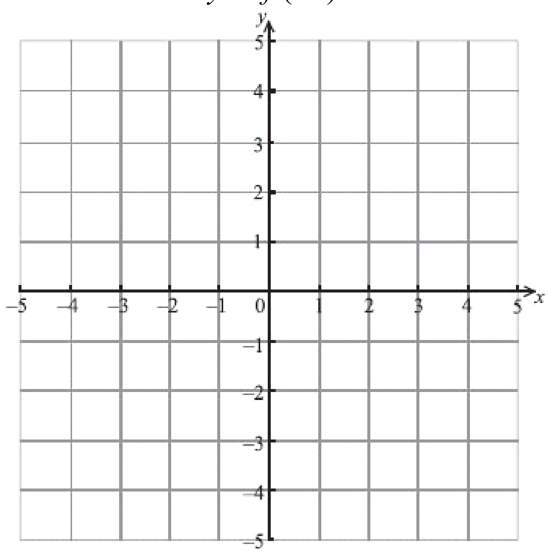
$y = 3f(-x)$



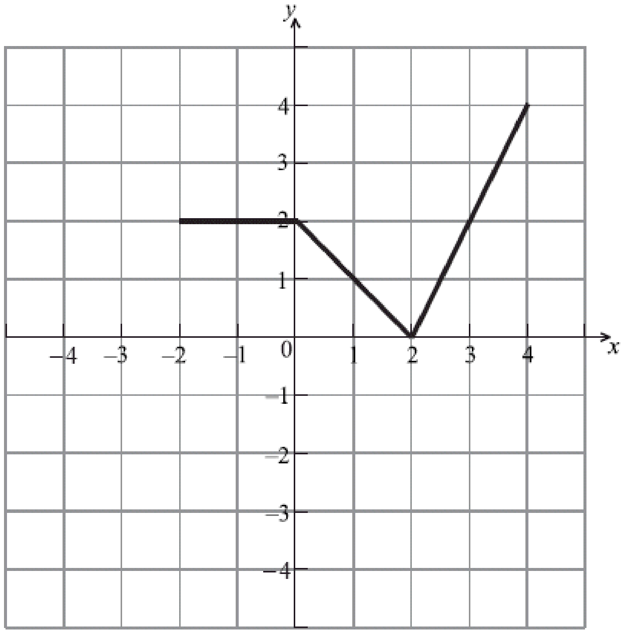
$y = f(2x)$



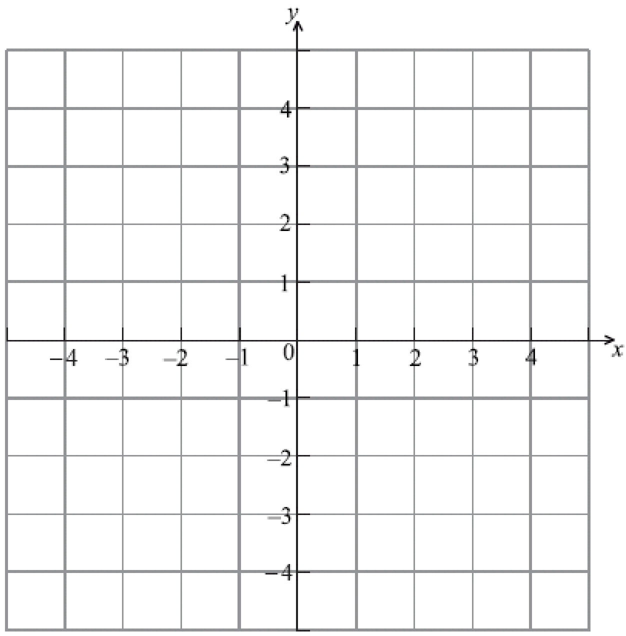
	$y = f(x)$	$y = 3f(-x)$	$y = f(2x)$
Domain	$0 \le x \le 4$		
Range	$0 \le y \le 1$		

8. [Maximum mark: 5]

The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 4$.



(a) Let $h(x) = f(-x)$. Sketch the graph of h on the grid below.



[2]

(b) Let $g(x) = \frac{1}{2}f(x-1)$. The point $\mathrm{A}(3, 2)$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P. [3]

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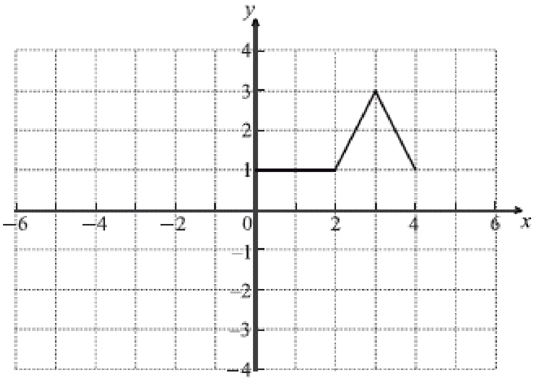
9. [Maximum mark: 6]

Consider the graph of f shown below.



(a) On the **same** grid sketch the graph of $y = f(-x)$. [2]

The following four diagrams show **images** of f under different transformations.

Diagram A

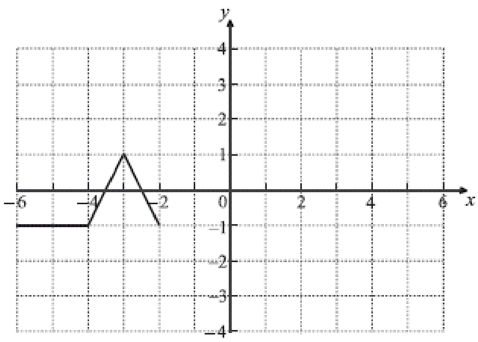


Diagram B

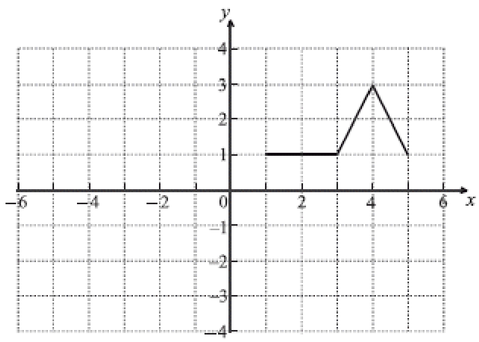


Diagram C

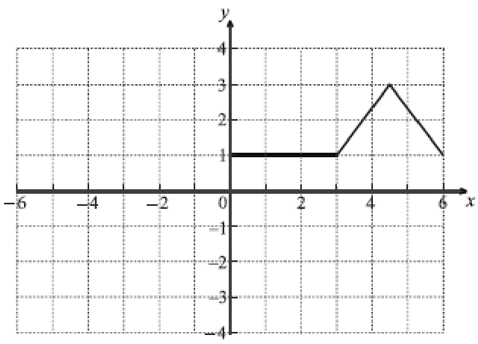
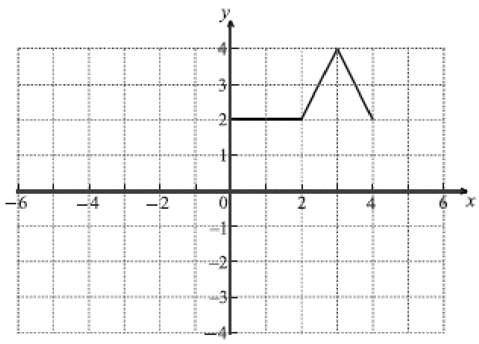


Diagram D



(b) Complete the following table. [2]

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	
Maps $f(x)$ to $f(x)+1$	

(c) Give a full geometric description of the transformation in Diagram A. [2]

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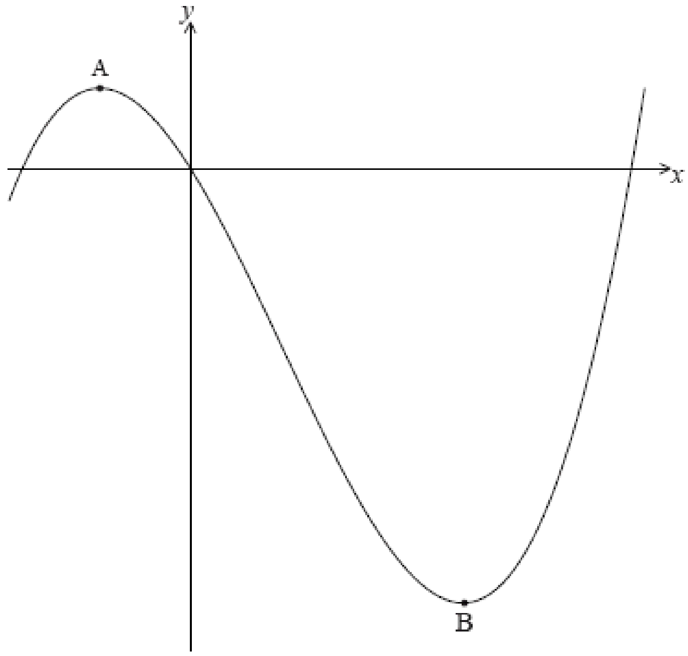
10. [Maximum mark: 8]

Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of *f* is shown below.



There is a maximum point at A and a minimum point at B(3, –9).

(a) Write down the coordinates of A. [2]

(b) Write down the coordinates of

(i) the image of B after reflection in the *y*-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the *x*-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$. [6]

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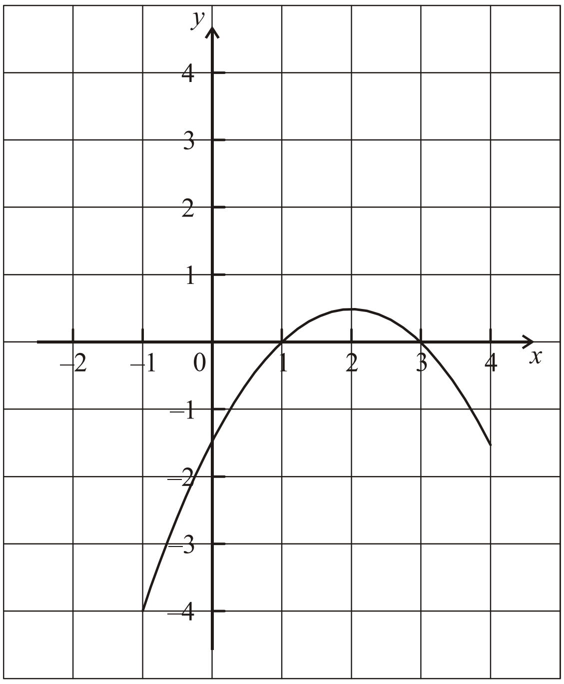
11. [Maximum mark: 10]

The graph of a function f is shown in the diagram below.



(a) On the same diagram sketch the graph of $y = -f(x)$. [2]

(b) Let $g(x) = f(x+3)$.

(i) Find $g(-3)$

(ii) Describe **fully** the transformation that maps the graph of f to the graph of g [4]

(c) Write down in the table below the domain and the range for each function:

	$y = f(x)$	$y = -f(x)$	$y = f(x+3)$
Domain	$-1 \leq x \leq 4$		
Range	$-4 \leq y \leq 0.5$		

[4]

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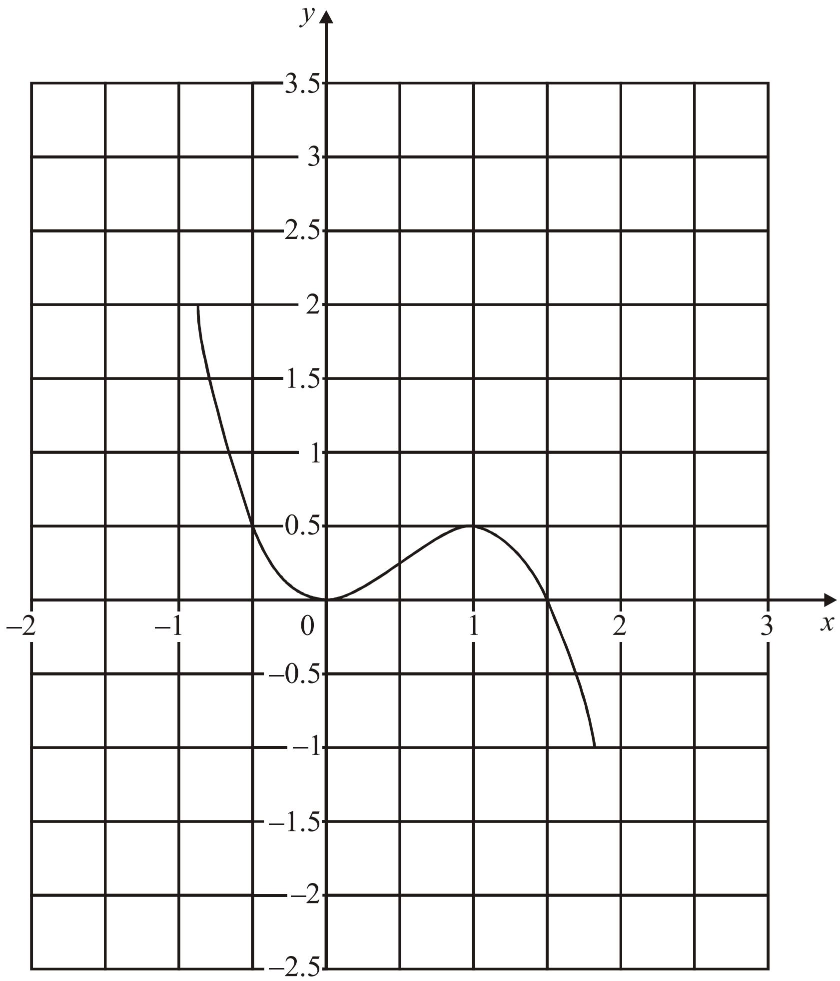
12. [Maximum mark: 4]

The following diagram shows the graph of $y = f(x)$. It has minimum and maximum points at (0, 0) and $(1, \frac{1}{2})$.



(a) On the same diagram, draw the graph of $y = f(x-1) + \frac{3}{2}$.

(b) What are the coordinates of the minimum and maximum points of $y = f(x-1) + \frac{3}{2}$?

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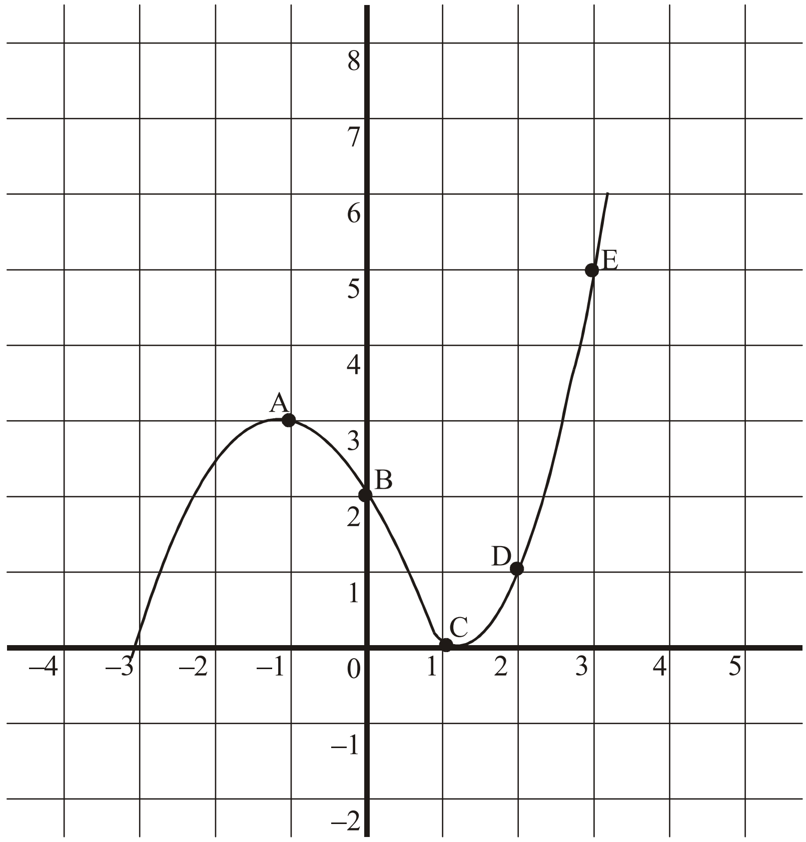
13. [Maximum mark: 6]

The sketch shows part of the graph of $y = f(x)$ which passes through the points A(–1, 3), B(0, 2), C(l, 0), D(2, 1) and E(3, 5).



A second function is defined by $g(x) = 2f(x - 1)$.

(a) Calculate $g(0)$, $g(1)$, $g(2)$ and $g(3)$.

(b) On the same axes, sketch the graph of the function $g(x)$.

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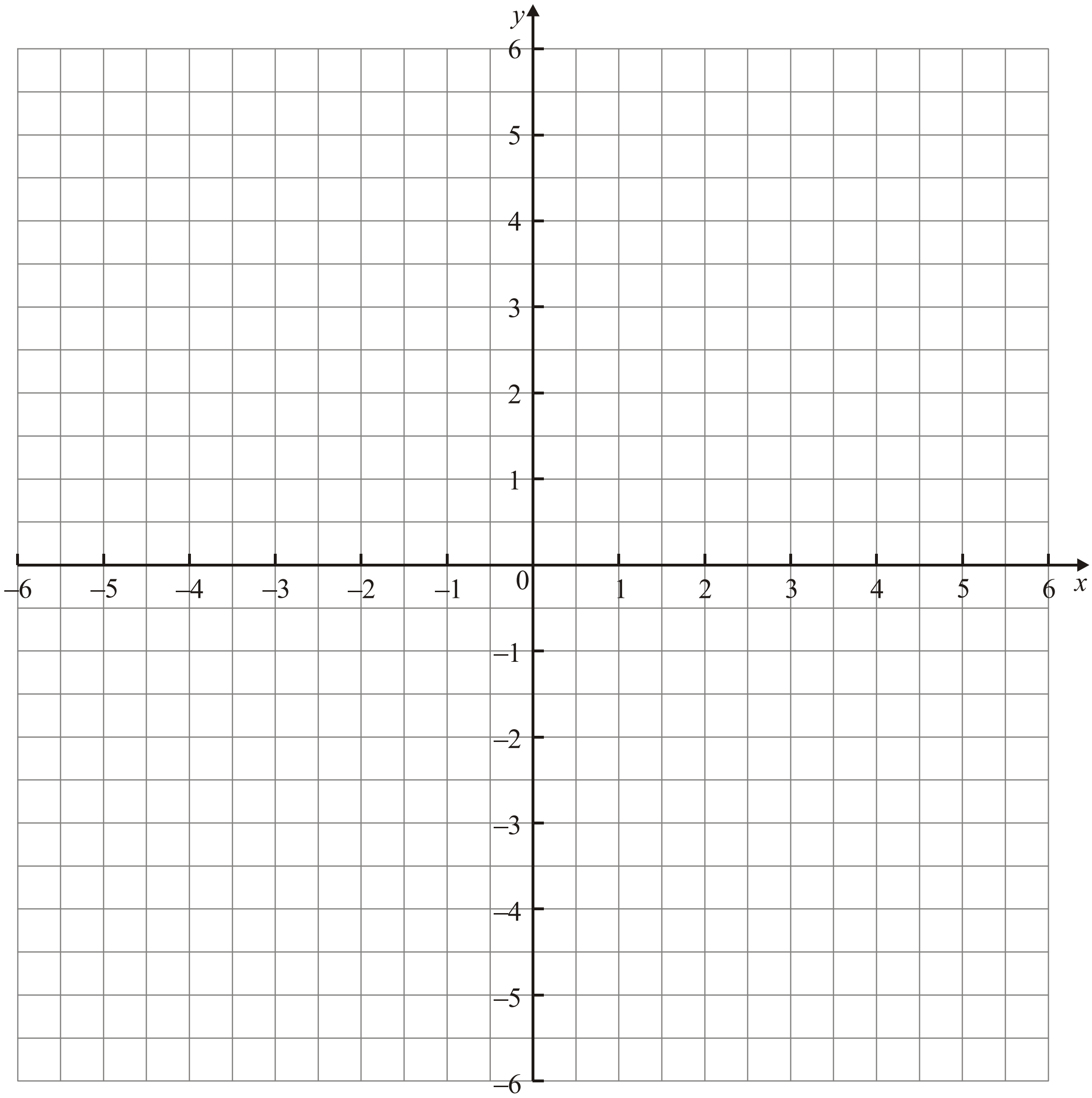
14. [Maximum mark: 8]

Let $f(x) = 2x + 1$.

(a) On the grid below draw the graph of $f(x)$ for $0 \leq x \leq 2$. [2]

(b) Let $g(x) = f(x+3) - 2$. On the grid below draw the graph of $g(x)$ for $-3 \leq x \leq -1$. [4]



(c) Write down the range for each function:

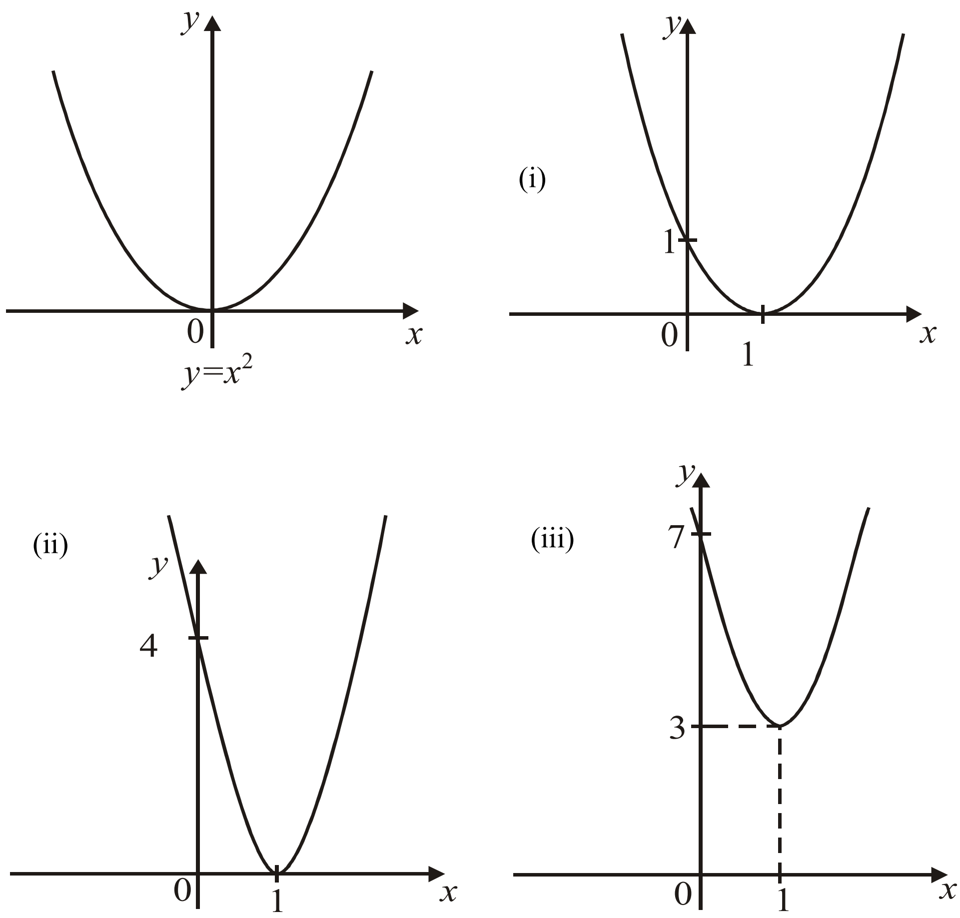
	$y = f(x)$	$y = g(x)$
Domain	$0 \leq x \leq 2$	$-3 \leq x \leq -1$
Range		

[2]

15. [Maximum mark: 6]

The diagrams show how the graph of $y = x^2$ is transformed to the graph of $y = f(x)$ in three steps.

(a) For each diagram give the equation of the curve.



[4]

(b) Write down the coordinates of the vertex at each step

	Equation of the curve	Vertex
	$y = x^2$	(0,0)
(i)		
(ii)		
(iii)		

[2]

16. [Maximum mark: 6]

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

(a) The graph of g can be obtained from the graph of f using two transformations. Give a full geometric description of each of the two transformations. [2]

(b) The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h.

The point (–1, 1) on the graph of f is translated to the point P on the graph of h.

Find the coordinates of P. [4]

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17. [Maximum mark: 6]

Let $f(x) = 3x^2$. The graph of *f* is translated 1 unit to the right and 2 units down.

The graph of g is the image of the graph of f after this translation.

(a) Write down the coordinates of the vertex of the graph of g. [2]

(b) Express g in the form $g(x) = 3(x-p)^2 + q$. [2]

The graph of h is the reflection of the graph of g in the x-axis.

(c) Write down the coordinates of the vertex of the graph of h. [2]

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18. [Maximum mark: 6]

(a) Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x - c)^2 + d$. [3]

The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the transformations

a vertical stretch with scale factor k **followed by**
a horizontal translation of p units **followed by**
a vertical translation of q units.

(b) Write down the value of (i) k; (ii) p; (iii) q. [3]

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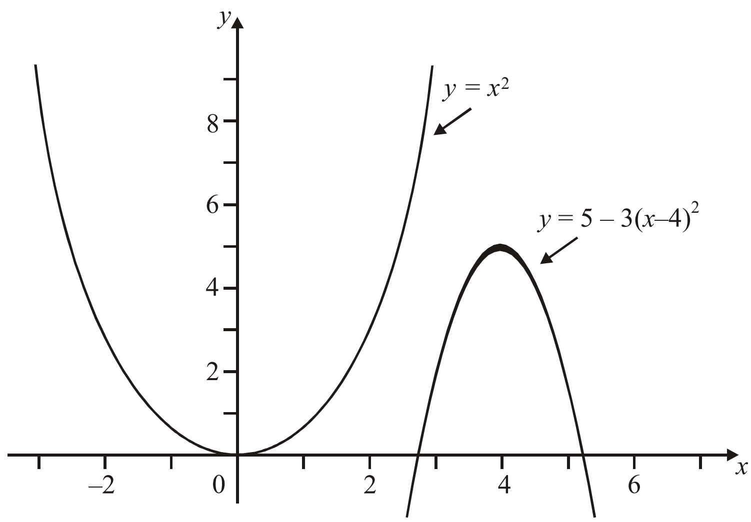
19. [Maximum mark: 4]

The diagram shows parts of the graphs of $y = x^2$ and $y = 5 - 3(x - 4)^2$.



The graph of $y = x^2$ may be transformed into the graph of $y = 5 - 3(x - 4)^2$ by these transformations.

A reflection in the line $y = 0$ **followed by**
a vertical stretch with scale factor k **followed by**
a horizontal translation of p units **followed by**
a vertical translation of q units.

Write down the value of (a) k (b) p (c) q

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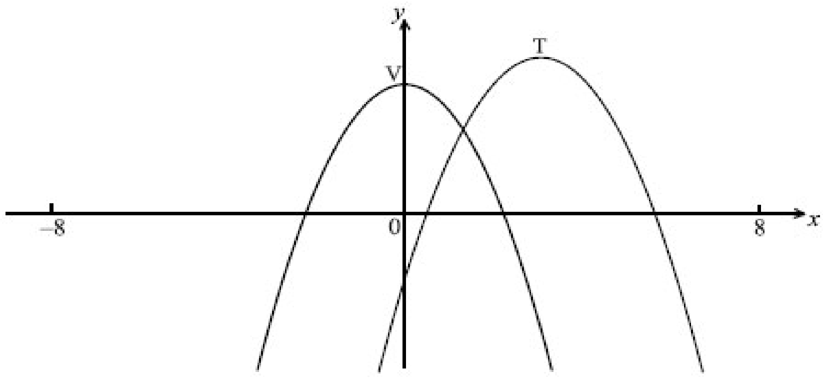
20. [Maximum mark: 6]

The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex V (0, 5).

Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex T (3, 6).



(a) Write down the value of (i) h (ii) k [2]

(b) Write down an expression for $g(x)$. [2]

(c) On the same diagram, sketch the graph of $y = g(-x)$ [2]

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21. [Maximum mark: 6]

The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

(a) Write f in the form $f(x) = 3(x-h)^2 + k$. [3]

(b) The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x-p)^2 + q$. [3]

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B. Paper 2 questions (*LONG*)

22. [Maximum mark: 12]

Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$. [2]

The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

(b) Find the coordinates of the vertex of the graph of h. [3]

(c) Show that $h(x) = x^2 - 8x + 19$. [2]

(d) The line $y = 2x - 6$ is a tangent to the graph of h at the point P.
Find the x-coordinate of P. [2]

(e) The line $y = 2x - 5$ intersects the graph of h at two points. Find the coordinates of the two points of intersection. [3]

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23. [Maximum mark: 15]

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. [2]

(b) For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the y-intercept;

(iv) find both x-intercepts. [8]

(c) **Hence** sketch the graph of f . [2]

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y-direction

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t . [3]

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