

INTERNATIONAL BACCALAUREATE  
*Mathematics: applications and interpretation*

**MAI**

**EXERCISES [MAI 1.4-1.5]**  
**ARITHMETIC SEQUENCES**  
*Compiled by Christos Nikolaidis*

**A. Paper 1 questions (SHORT)**

**SEQUENCES (IN GENERAL)**

1. [Maximum mark: 5]

Consider the sequence  $u_n$  :                    2, 5, 10, 3, 7, 4, ....

(notice that there is no particular pattern in this sequence)

(a) Write down the values of  $u_1$  and  $S_1$ . [2]

(b) Write down the values of  $u_2$  and  $S_2$ . [2]

(c) Find  $S_5$ . [1]

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2. [Maximum mark: 5]

Consider the (simplest) sequence  $u_n$  :                    1, 2, 3, 4, 5, 6, ....

(a) Write down the values of  $u_2$  and  $u_{20}$ . [2]

(b) Find the value of  $S_5$ . [2]

(c) Express the value of  $u_n$  in terms of  $n$ . [1]

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3. [Maximum mark: 6]

Consider the sequence  $u_n$  given by  $u_n = 10n$

- (a) Find the first three terms . [2]
- (b) Find  $u_{10}$  [1]
- (c) Find  $S_1$ ,  $S_2$  and  $S_3$ . [3]

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4. [Maximum mark: 6]

Consider the sequence  $u_n$  given by  $u_n = 10n^2$

- (a) Find the first three terms . [2]
- (b) Find  $u_{10}$  [1]
- (c) Find  $S_1$ ,  $S_2$  and  $S_3$ . [3]

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5. [Maximum mark: 6]

Consider the sequence  $u_n$  given by  $u_n = 10^n$

- (a) Find the first three terms . [2]
- (b) Find  $u_{10}$  [1]
- (c) Find  $S_1$ ,  $S_2$  and  $S_3$ . [3]

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6. [Maximum mark: 6]

Consider the sequence  $u_n$  given by the recursive definition

$$u_1 = 10, \quad u_{n+1} = u_n + 10$$

(a) Find the first three terms . [3]

(b) Find  $S_1$ ,  $S_2$  and  $S_3$ . [3]

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7. [Maximum mark: 6]

Consider the sequence  $u_n$  given by the recursive definition

$$u_1 = 10, \quad u_{n+1} = 2u_n + 10$$

(a) Find the first three terms . [3]

(b) Find  $S_1$ ,  $S_2$  and  $S_3$ . [3]

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11. [Maximum mark: 6]

(a) Write down the first three terms of the sequence  $u_n = 3n$ , for  $n \geq 1$ . [1]

(b) Find (i)  $\sum_{n=1}^{20} 3n$ ; (ii)  $\sum_{n=21}^{100} 3n$ . [5]

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12. [Maximum mark: 6]

Let  $u_n = 3 - 2n$ .

(a) Write down the value of  $u_1$ ,  $u_2$ , and  $u_3$ . [3]

(b) Find  $\sum_{n=1}^{20} (3 - 2n)$ . [3]

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15. [Maximum mark: 6]

In an arithmetic sequence,  $u_1 = 2$  and  $u_3 = 8$ .

- (a) Find  $d$ . [2]
- (b) Find  $u_{20}$ . [2]
- (c) Find  $S_{20}$ . [2]

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16. [Maximum mark: 4]

In an arithmetic sequence, the first term is 5 and the fourth term is 40. Find the second term.

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19. [Maximum mark: 6]

The  $n^{\text{th}}$  term of an arithmetic sequence is given by  $u_n = 5 + 2n$ .

(a) Write down the common difference. [1]

(b) (i) Given that the  $n^{\text{th}}$  term of this sequence is 115, find the value of  $n$ .

(ii) For this value of  $n$ , find the sum of the sequence. [5]

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20. [Maximum mark: 6]

An arithmetic sequence,  $u_1, u_2, u_3, \dots$ , has  $d = 11$  and  $u_{27} = 263$ .

(a) Find  $u_1$ . [2]

(b) (i) Given that  $u_n = 516$ , find the value of  $n$ .

(ii) For this value of  $n$ , find  $S_n$ . [4]

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23. [Maximum mark: 4]

Consider the arithmetic series  $2 + 5 + 8 + \dots$

(a) Find an expression for  $S_n$ , the sum of the first  $n$  terms. [2]

(b) Find the value of  $n$  for which  $S_n = 1365$ . [2]

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24. [Maximum mark: 6]

Let  $S_n$  be the sum of the first  $n$  terms of an arithmetic sequence, whose first three terms are  $u_1, u_2$  and  $u_3$ . It is known that  $S_1 = 7$ , and  $S_2 = 18$ .

(a) Write down  $u_1$ . [1]

(b) Calculate the common difference [3]

(c) Calculate  $u_4$ . [2]

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26. [Maximum mark: 6]

In an arithmetic sequence, the first term is  $-2$ , the fourth term is  $16$ , and the  $n^{\text{th}}$  term is  $11\,998$ .

(a) Find the common difference  $d$ . [3]

(b) Find the value of  $n$ . [3]

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27. [Maximum mark: 5]

In an arithmetic sequence  $u_1 = 7$ ,  $u_{20} = 64$  and  $u_n = 3709$ .

(a) Find the value of the common difference. [3]

(b) Find the value of  $n$ . [2]

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29. [Maximum mark: 7]

In an arithmetic sequence  $u_{21} = -37$  and  $u_4 = -3$ .

(a) Find (i) the common difference; (ii) the first term. [4]

(b) Find  $S_{10}$ . [3]

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30. [Maximum mark: 5]

In an arithmetic series, the first term is  $-7$  and the sum of the first 20 terms is 620.

(a) Find the common difference. [3]

(b) Find the value of the 78<sup>th</sup> term. [2]

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31. [Maximum mark: 6]

In an arithmetic sequence,  $S_{40} = 1900$  and  $u_{40} = 106$ . Find the value of  $u_1$  and of  $d$ .

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32. [Maximum mark: 6]

In an arithmetic sequence, let  $S_1=10$ , and  $S_2=25$

- (a) Write down  $u_1$  and  $u_2$ . [2]
- (b) Find the common difference  $d$ . [1]
- (c) Find  $S_3$  and  $S_4$ . [2]
- (d) Express  $S_n$  in terms of  $n$ . [1]

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35. [Maximum mark: 5]

Consider the arithmetic series  $-6 + 1 + 8 + 15 + \dots$

Find the least number of terms so that the sum of the series is greater than 10 000.

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36. [Maximum mark: 5]

The first three terms of an arithmetic sequence are  $a + 3$ ,  $2a + 4$  and  $a + 9$ .

- (a) Find the value of  $a$ . [3]
- (b) Confirm the result. [2]

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**PROBLEMS**

**37.** [Maximum mark: 5]

Gwendolyn added the multiples of 3, from 3 to 3750 and found that

$$3 + 6 + 9 + \dots + 3750 = s.$$

Calculate  $s$ .

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**38.** [Maximum mark: 6]

Arturo goes swimming every week. He swims 200 metres in the first week.

Each week he swims 30 metres more than the previous week. He continues for one year (52 weeks).

- (a) How far does Arturo swim in the final week? [3]
- (b) How far does he swim altogether? [3]

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39. [Maximum mark: 6]

A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.

- (a) Calculate the number of seats in the 20<sup>th</sup> row. [4]
- (b) Calculate the **total** number of seats. [2]

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40. [Maximum mark: 4]

Each day a runner trains for a 10 km race. On the first day she runs 1000 m, and then increases the distance by 250 m on each subsequent day.

- (a) On which day does she run a distance of 10 km in training? [2]
- (b) What is the total distance she will have run in training by the end of that day? Give your answer exactly. [2]

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41. [Maximum mark: 6]

An 81 metre rope is cut into  $n$  pieces of increasing lengths that form an arithmetic sequence with a common difference of  $d$  metres. Given that the lengths of the shortest and longest pieces are 1.5m and 7.5m respectively, find the values of  $n$  and  $d$

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42. [Maximum mark: 4]

A woman deposits \$100 into her son's savings account on his first birthday. On his second birthday she deposits \$125, \$150 on his third birthday, and so on.

- (a) How much money would she deposit into her son's account on his 17th birthday? [2]
- (b) How much in total would she have deposited after her son's 17th birthday? [2]

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43. [Maximum mark: 6]

A teacher earns an annual salary of 45 000 USD for the first year of her employment  
Her annual salary increases by 1750 USD each year.

(a) Calculate the annual salary for the fifth year of her employment. [3]

She remains in this employment for 10 years.

(b) Calculate the **total** salary she earns in this employment during these 10 years. [3]

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44. [Maximum mark: 6]

A concert choir is arranged, per row, according to an arithmetic sequence. There are 20  
singers in the fourth row and 32 singers in the eighth row.

(a) Find the common difference of this arithmetic sequence. [3]

There are 10 rows in the choir and 11 singers in the first row.

(b) Find the **total** number of singers in the choir. [3]

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