# EXERCISES [MAI 4.12]

# POISSON DISTRIBUTION

# SOLUTIONS

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## A. Paper 1 questions (SHORT)

**1.** (a)

| | | | |
|---|---|---|---|
| $P(X=3)$ | 0.21376.. ≅ 0.214 | $P(3 \le X \le 5)$ | 0.41416.. ≅ 0.414 |
| $P(X \le 3)$ | 0.75757.. ≅ 0.758 | $P(X \ge 3)$ | 0.45618.. ≅ 0.456 |
| $P(X < 3)$ | 0.54381.. ≅ 0.544 | $P(X > 3)$ | 0.24242.. ≅ 0.242 |

(b)

| | |
|---|---|
| $P(X=3 \mid X \ge 3)$ | 0.21376/0.45618 = 0.469 |
| $P(X \le 5 \mid X \ge 3)$ | 0.41416/0.45618 = 0.908 |
| $P(X \ge 5 \mid X \ge 3)$ | 0.10882/0.45618 = 0.239 |
| $P(X \le 3 \mid X \le 5)$ | 0.75757/0.95797 = 0.791 |

(c) $P(X=2) = 0.257$ and $P(X=3) = 0.214$, hence **mode** = 2

**2.** (a) For $m = 2$ $P(X=1) = 0.271$

(b) For $m = 2$ $P(X=2) = 0.271$

(c) For $m = 4$ $P(X=1) = 0.0733$

(d) For $m = 4$ $P(X=2) = 0.147$

(e) For $m = 1$ $P(X=0) = 0.368$

**3.** (a) For $m = 1$ $P(X=1) = 0.368$

(b) For $m = 0.5$ $P(X=1) = 0.303$

(c) For $m = 0.5$ $P(X=0) = 0.607$

(d) For $m = 2$ $P(X=3) = 0.180$

**4.** (a) For $m = 2$ $P(X=3) = 0.180$

(b) For $m = 2$ $P(X \le 3) = 0.857$

(c) For $m = 2$ $P(X \ge 3) = 0.323$

**5.** $X$ follows Poisson distribution with $m = 2$

| *Bet* | 10€ | -1€ | -2€ |
|---|---|---|---|
| $X$ | 0 | 1 or 2 | ≥3 |
| Probability | 0.135 | 0.541 | 0.323 |

E(*Profit*) = 0.163. For 10 times Expected profit = 10×0.163 = 1.63€

**6.** (a) For $m = 0.5$ $P(X = 0) = 0.60653\ldots \cong 0.607$

(b) For $m = 2.5$ $P(X = 0) = 0.08208\ldots \cong 0.0821$

(c) Binomial $Y$ with $n = 5$, $p = 0.60653$, $P(Y = 2) = 0.224$

**7.** (a) $P(3 \le X \le 5) = P(X \le 5) - P(X \le 2) = 0.547$

(b) $P(X \ge 3) = 1 - P(X \le 2) = 0.762$

(c) $P(3 \le X \le 5 | X \ge 3) = \dfrac{P(3 \le X \le 5)}{P(X \ge 3)}\left(= \dfrac{0.547}{0.762}\right) = 0.718$

**8.** $P(X \ge 5) = 0.1847$

**9.** (a) Let $X$ denote the number of flaws in one metre of the wire. $m = 2.3$

$P(X = 2) = 0.265.$

(b) Let $Y$ denote the number of flaws in two metres of wire $m = 2 \times 2.3 = 4.6$

$P(Y \ge 1) = 0.990$ (3 s.f.)

**10.** (a) $m = 1.5$ $P(X > 2) = 0.191$

(b) $m = 1.5$ $P(X > 2 | X \ge 1) = \dfrac{0.191153}{0.776869} = 0.246$

**11.** (a) $m = 0.6$ $P(X \ge 2) \cong 0.122$

(b) $P(X = 0) = 0.548 > 0.5$ so $X = 0$ is the mode

(c) $m = 7 \times 0.6 = 4.2$ $P(Y = 0) \cong 0.0150$

(**OR** with $m = 0.6$ $P(X = 0) = 0.548$ .. so $0.548^7 \cong 0.0150$

**12.** (a) $X \sim Po(3.2)$ $P(X = 4) \cong 0.178$

(b) $Y \sim Po(1.90)$ $P(Y = 3) \cong 0.171$

Required probability $= 0.171 \times 0.178 \cong 0.0304$

**13.** (a) $m = 4$ $P(X + Y = 3) = 0.19536.. \cong 0.195$

(b)

| $a$ | $b$ | $P(X = a)$ | $P(Y = b)$ | $P(X = a \text{ and } Y = b)$ |
|---|---|---|---|---|
| 0 | 3 | 0.04979 | 0.06131 | 0.00305 |
| 1 | 2 | 0.14936 | 0.18394 | **0.02747** |
| 2 | 1 | **0.22404** | **0.36788** | **0.08242** |
| 3 | 0 | **0.22404** | 0.36788 | 0.08242 |

(c) sum = 0.00305+ 0.02747+ 0.08242 + 0.08242 = 0.19536 .. $\cong$ 0.195

The result is equal to the result in question (a).

Both results calculate the probability that $X + Y = 3$

**14.** (a) $X + Y \sim \text{Po}(4)$, $P(X+Y = 3) = 0.19536.. \cong 0.195$

(b) $$P(X = 1 \text{ and } Y = 2 \mid X + Y = 3) = \frac{0.02747}{0.19536} = 0.141$$

(c) $$P(X < Y \mid X + Y = 3) = \frac{0.00305 + 0.02747}{0.19536} = 0.156$$ (see table in question 13)

**15.** $A + B \sim \text{Po}(7)$,

(a) $P(A+B = 5) = 0.12771.. \cong 0.128$

(b) $P(A+B < 5) = 0.17299.. \cong 0.173$

(c) $P(A+B > 5) = 0.69929.. \cong 0.699$

**16.** $A \sim \text{Po}(4)$, $B \sim \text{Po}(5)$, so $A + B \sim \text{Po}(9)$,

(a) $P(A = 4 \text{ and } B = 5) = 0.19536.. \times 0.17546.. \cong 0.0343$

(b) $P(A + B = 10) \cong 0.119$

(c) $m = 7\times9 = 63$, $P(A + B \leq 50) \cong 0.0537$

**17.** $B \sim \text{Po}(2.7)$, $S \sim \text{Po}(2.5)$

(a) (i) $P(B = 2) = 0.245$

(ii) $P(S = 3) = 0.214$

(iii) The two events are independent. $P(B=2)\times P(S=3) = 0.214\times0.245 \cong 0.0524$

(b) $B + S \sim \text{Po}(5.2)$, $P(B + S = 5) = 0.17478.. \cong 0.175$

(c) $P(B = 2)\, P(S = 3) = 0.245 \times 0.214 \cong 0.0524$

$P(B = 1)\, P(S = 4) = 0.181\times0.133 \cong 0.0242$

$P(B = 0)\, P(S = 5) = 0.067\times0.067 \cong 0.0045$

$$P(B < S) = \frac{0.0524 + 0.0242 + 0.0045}{0.175} = \frac{0.0811}{0.175} \cong 0.464 \text{ (or } 0.463)$$

**18.** (a) For a 15 minute period $m = 15/4 = 3.75$. $P(X = 6)$ 0.0908

(b) For the first operator $m_1 = 0.01 \times 20 = 0.2$

For the second operator $m_2 = 0.03 \times 40 = 1.2$

(c) Let $F_1$, $F_2$ be random variables which represent the number of failures to answer telephone calls by the first and the second operator, respectively.

$F_1 \sim P_0\,(0.2)$ and $F_2 \sim P_0\,(1.2)$.

Since $F_1$ and $F_2$ are independent $F_1 + F_2 \sim P_0\,(0.2 + 1.2) = P_0(1.4)$

$P(F_1 + F_2 \geq 2) = 0.408$

## B. Paper 2 questions (LONG)

**19.** (a) For 4 weeks $m = 2\times4 = 8$, $P(X = 8) = 0.13958\ldots \cong 0.140$

(b) $m = 8$, $P(X > 8) = 0.40745 \ldots \cong 0.407$

(c) $m = 8$, $P(X \geq 8) = 0.54703\ldots \cong 0.547$

(d) $Y \sim B(n,p)$ with $n = 13$, $p = 0.54703$, $P(X > 9) = 0.0894$

**20.** (a) mean for 30 days: $30 \times 0.2 = 6$. $P(X = 4) = 0.134$

(b) $P(X > 3) = 0.849$

(c) **EITHER**

mean for five days: $5 \times 0.2 = 1$ $P(X = 0) = 0.368$

**OR**

mean for one day: 0.2 $P(X = 0) = 0.81873$

so for 5 days $0.81873^5 = 0.368$

(d) Required probability $= 0.81873 \times 0.81873 \times (1 - 0.81873) = 0.122$

(e) Expected cost is $1850 \times 6 = 11100$ euros

(f) On any one day $P(X = 0) = 0.81873$

$Y \sim B(5, 0.81873)$, $P(Y = 4) = 0.407$

**21.** (a) (i) $P(4.8 < X < 7.5) = 0.629$

(ii) $P(X < d) = 0.15$, $d = 4.45$ (km)

(b) (i) $P(T \geq 3) = 0.679...$

Required probability is $(0.679...)^2 = 0.461$

(ii) $m = 3.5 \times 5 = 17.5$ $P(Y = 15) = 0.0849$

**22.** (a) $P(T < 10) = 0.299$, 30%

(b) $m = 2\times6 = 12$, $P(X = 8) = 0.0655$

(c) $m = 12$, $P(X = 13) = 0.1055$, $P(X \geq 10) = 0.7576$

$$P(X = 13 \mid X \geq 10) = \frac{P(X = 13)}{P(X \geq 10)} = \frac{0.1055}{0.7576} = 0.139$$

(d) Let $Y$ be the random variable "number of text messages in 15 minutes"
Let $D$ be the random variable "number of days with no messages received"

$Y \sim \text{Po}(1.5)$

$P(Y = 0) = 0.2231...$

$D \sim B(5, 0.2231...)$

$P(D = 3) = 0.0670$