EXERCISES [MAI 4.11]

NORMAL DISTRIBUTION

SOLUTIONS

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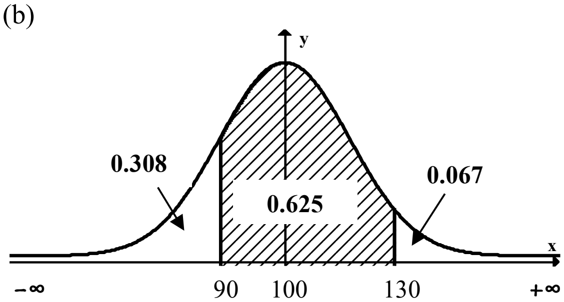
A. Paper 1 questions (SHORT)

1. (a) $P(X < 90) = 0.308$ $P(90 < X < 130) = 0.625$ $P(X > 130) = 0.067$

(b)



(c) (Standardized values) $m = -0.5$, $n = 1.5$

2. (a) (use tail left, area 0.8) $a = 116.8$

(b) (use tail right, area 0.3) b=110.5

(c) (use tail central, area 0.5) $Q_1 = 86.5$ and $Q_3 = 113.5$

3. (a) $P(X < 130) = 0.8$

(b) $P(X < 70) = 0.2$

(c) $P(100 < X < 130) = 0.3$

(d) $P(70 < X < 130) = 0.6$

4. (a) $M \sim N(750, 625)$

(i) $P(M < 740 \text{ g}) = 0.345$

(ii) $P(M > 780 \text{ g}) = 0.115$

(iii) $P(740 < M < 780) = 0.540$

(b) P (both < 740) = $0.345^2 = 0.119$

(c) $x = 737$ g

5. (a) 0.773

(b) $d = 161$

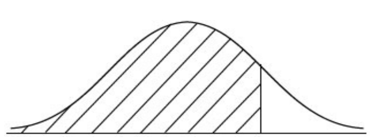
6. (a) 0.159

(b) $d = 136$

7. (a) 0.0668

(b) $k = 26.1$ kg

(c)



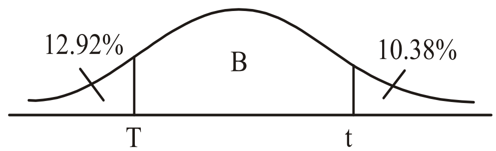
8. (a) $P(90 < X < 125) = 0.701$

70.1 percent of the population (accept 70 percent).

(b) $P(X \geq 125) = 0.0478$

P(both persons having IQ $\geq$ 125) = $(0.0478)^2 = 0.00228$

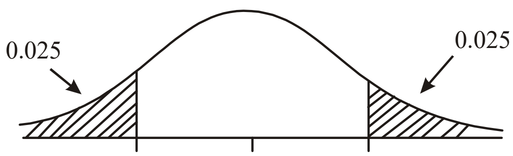
9. (a)



(b) $r = 6.56$ $t = 7.16$

10. (a) $P(M \geq 350) = 0.0912$

(b)



either tail left, tail right for each endpoint

or directly tail central with Area = 0.95

$251 < M < 369$

11. (a) 0.345 (34.5%)

(b) $P(2\,300 < X < 3\,300) = 0.645$

$P(\text{both}) = (0.645)^2 = 0.416$

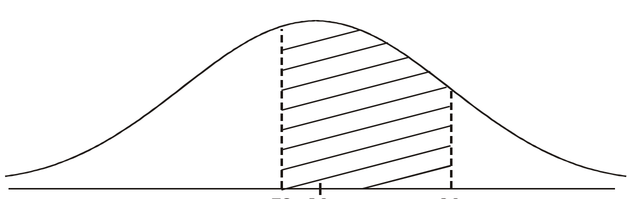
(c) $d = \$\,3\,325$ (= $3 330 to 3 s.f.) (Accept $3325.07)

12. $X \sim N(80, 8^2)$

(a) $P(X < 72) = 0.159$

(b) (i) $P(72 < X < 90) = 0.736$

(ii)



(c) $x = 66.0$ months

13. $\sigma = 10$, $1.12 \times 10 = 11.2$

(a) $100 + 11.2 = 111.2$

(b) $100 - 11.2 = 88.8$

14. (a) $p = 0.94$

(b) $D = 10$

(c) $P(17 < H < 24) = 0.44$

E(trees) = $200 \times 0.44 = 88$

15. (a) $P(H > 197) = 0.159 = 15.9\%$

(b) 99% of heights under 209.6 = 210 cm (3 sf)

Height of standard doorway = 210 + 17 = 227 cm

16. (a) $P(X = 102) = 0$

(b) $P(101.5 \le X < 102.5) = 0.0736$

(c) (i) $P(X > 102 \mid X > 100) = \dfrac{P(X > 102)}{P(X > 100)} = \dfrac{0.34457}{0.5} = 0.172$

(ii) $P(X < 102 \mid X > 100) = \dfrac{P(100 < X < 102)}{P(X > 100)} = \dfrac{0.15542}{0.5} = 0.0777$

17. (a) $P(X > 45) = 0.8943... \cong 0.894$

(b) $P(X > 55 \mid X > 45) = \dfrac{P(X > 55)}{P(X > 45)} = \dfrac{0.1056..}{0.8943..} = 0.118$

18. (a) $P(X > 350) = 0.10564977... \cong 0.106$

$1000 \times 0.106 = 106$

(b) (i) $(0.1056)^2 = 0.0112$

(ii) $0.1056 \times (1 - 0.1056) \times 2 = 0.189$

(iii) $0.0112 + 0.189 = 0.2$

(c) Binomial with $n = 8$ and $p = 0.1056$

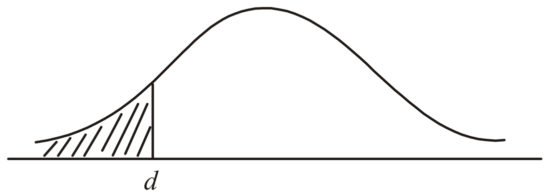
(i) $P(Y = 4) = 0.00557$ (ii) $P(Y \ge 1) = 0.591$

B. Paper 2 questions (LONG)

19. $X \sim N(7, 0.5^2)$

(a) (i) $P(X < 8) = 0.977$ (ii) $P(6 < X < 8) = 0.95$

(b) (i)



(ii) $d = 6.18$

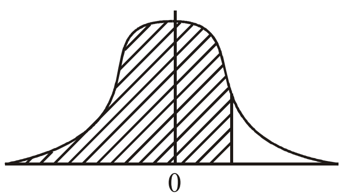
(c) IQR = Q3 - Q1 = 7.33724.. - 6.66275.. = 0.674

20. (a) Let X be the lifespan in hours, $X \sim N(57, 4.4^2)$

(i) $a = -0.455$ (3 sf) $b = 0.682$ (3 sf)

(ii) (a) $P(X > 55) = 0.675$ (b) $P(55 \le X \le 60) = 0.428$ (3 sf)

(b) 90% have died $\Rightarrow$ shaded area = 0.9

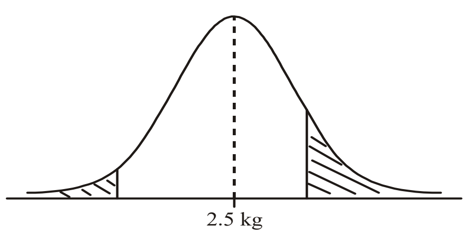


$t = 62.6$ hours

21. $W \sim N(2.5, 0.3^2)$

(a) (i) 0.048 (ii) 0.159

(iii)



(iv) $P = 0.7936$

(b) (i) $X \sim B(10, 0.7935...)$

$P(X = 10) = (0.7935...)^{10}$ **OR** $P(X = 10) = 0.0990$ (3 sf)

(ii) $X \sim B(10, 0.7935...)$

$P(X \ge 7) = 0.867$

22. Girls'height G ~ N(155,10^2), Boys'height B~ N(160,12^2)

(a) P(G > 170) = 0.0668

(b) x = 142

(c) r = 180 q = 140

(d) P(H > 170) = 0.60×0.0668 + 0.40×0.202 = 0.12088 = 0.121 (3 sf)

(e) $P(F \mid H > 170) = \dfrac{0.60 \times 0.0668}{0.121} = 0.332$

23. (a) B~N(1.63,0.16^2) , P(B < 1.52) = 0.24588… so 24.6%

(b) A~N(1.56,0.16^2) , P(A < 1.52) 0.40129

P(d < 1.52) = 0.44 × 0.40129 + 0.56 × 0.24588 = 0.314 (3 s.f.)

(c) $P\left(\text{bolt produced by B} \mid d < 1.52\right) = \dfrac{0.56 \times 0.24588}{0.31426} = 0.438$

(d) P(B >1.83) = 0.10564

$P(1.52 < B < 1.83)$ = 0. 64846

E(gain per bolt) = (0.24588)×(– 0.85) + (0. 64846)×(1.50) + (0.10564)×(0.50)= 0.81651

Expected gain = 8000 × 0.81651 = 6532 ($)

24. K = length of Karl's throw ~ N (59.50, 3.00^2)

I = length of Ian's throw. ~ N (60.33, 1.95^2)

(a) P(K > 56) = 0.8783 so 87.8%

(b) P(I > x) = 0.80 ⇒ x = 58.69 m

(c) (i) $Y \sim N(59.50, 3.00^2)$ $X \sim N(60.33, 1.95^2)$

P(K ≥ 65) = 0.0334 P(I ≥ 65) = 0.00831

Karl is more likely to qualify since P(Y ≥ 65) > P(X ≥ 65)

(ii) For both athletes Binomial with n = 3, p_K = 0.0334 p_I = 0.00831

P (Karl qualifies) = P(X_K ≥ 3) = 0.0969

P(Ian qualifies) = P(X_I ≥ 3) = 0.0247

P(both qualify) = (0.0969) (0.0247) = 0.00239