EXERCISES [MAI 3.5]

ARCS AND SECTORS

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. (a) $l_{MINOR} = \frac{60}{360}2\pi(10) = \frac{10\pi}{3}$ $(\cong 10.5)$ $l_{MAJOR} = \frac{300}{360}2\pi(10) = \frac{50\pi}{3}$ $(\cong 52.4)$

(b) $A_{MINOR} = \frac{60}{360}\pi(10)^2 = \frac{50\pi}{3}$ $(\cong 52.4)$ $A_{MAJOR} = \frac{300}{360}\pi(10)^2 = \frac{250\pi}{3}$ $(\cong 262)$

(c) $P_{MINOR} = l_{MINOR} + 2r = \frac{10\pi}{3} + 20$ $(\cong 30.5)$ $P_{MAJOR} = l_{MAJOR} + 2r = \frac{50\pi}{3} + 20$ $(\cong 72.4)$

2. (a) $l_{ACB} = \frac{90}{360}2\pi(15) = \frac{15\pi}{2}$ $(\cong 23.6)$

(b) $A\hat{O}B$ (obtuse) = 360-90 = 270

Area = $\frac{270}{360}\pi(15)^2 = \frac{675\pi}{4}$ $(\cong 530)$

3. (a) **METHOD 1**

cosine rule: AB = $\sqrt{3.9^2 + 3.9^2 - 2(3.9)(3.9)\cos 100}$ = 5.98(cm)

METHOD 2

using right-angled triangles: sin 50 = $\frac{x}{3.9} \Rightarrow x = 3.9 \sin 50$

AB = $2x$ = 5.98 (cm)

METHOD 3

sine rule: $\frac{\sin 40}{3.9} = \frac{\sin 100}{AB} \Rightarrow$ AB = 5.98 (cm)

(b) $A\hat{O}B = 2\pi - 1.8$ (= 4.4832)

$A = \frac{260}{360}\pi(3.9)^2 = \frac{2197\pi}{200}$ $(\cong 34.5\ (\text{cm}^2))$

4. (a) $l_{ACB} = \frac{60}{360}2\pi(10) = \frac{10\pi}{3}$ $(\cong 10.5)$

(b) area of large sector = $\frac{60}{360}\pi(10^2) = \frac{50\pi}{3}$, area of small sector = $\frac{60}{360}\pi(8^2) = \frac{32\pi}{3}$

area shaded $= \frac{18\pi}{3} = 6\pi$ $(\cong 18.8)$

(c) length of small arc = $\frac{60}{360}2\pi(8) = \frac{8\pi}{3}$

Perimeter = $\frac{10\pi}{3} + \frac{8\pi}{3} + 2 + 2 = 6\pi + 4$ $(\cong 22.8)$

5. $\frac{\theta}{360}\pi(5.4)^2 = 21.6 \Rightarrow \theta = 84.882536...$

arc $AB = \frac{84.882536...}{360} 2\pi(5.4) = 8$ cm

6. (a) $\frac{70}{360}\pi r^2 = 27 \quad \Rightarrow r = 6.6482780... \simeq 6.65$

(b) arc $ACB = \frac{70}{360} 2\pi(6.6482780...) \cong 8.12$

7. (a) $\frac{40}{360} 2\pi r = 3\pi \Rightarrow r = \frac{27}{2} = 13.5$

(b) perimeter $= 2r + l = 27 + 3\pi$ (cm) ($\cong 36.4$)

(c) area $= \frac{40}{360}\pi(13.5)^2 \cong 63.6$

8. $\frac{\theta}{360}\pi r^2 = 180$

$\frac{\theta}{360} 2\pi r = 24$

Divide the two relations: $\frac{\pi r^2}{2\pi r} = \frac{180}{24} \Leftrightarrow r = 15$

$\frac{\theta}{360} 2\pi \times 15 = 24 \Rightarrow \theta = \frac{288}{\pi}$ ($\cong 91.7°$)

9. (a) $\frac{120}{360}\pi \times 15^2 = 75\pi$ ($\cong 236$)

(b) Area $\Delta OAB = \frac{1}{2} 15^2 \sin 120° = 97.42785...$ ($\cong 97.4$)

(c) Area $= 75\pi - 97.42785... = 138$

(d) $AB^2 = 15^2 + 15^2 - 2 \times 15 \times 15 \cos 120° \Rightarrow AB = 15\sqrt{3}$ ($\cong 26.0$)

Arc AB $= \frac{120}{360} 2\pi \times 15 = 10\pi$ ($\cong 31.4$)

Perimeter $= 15\sqrt{3} + 10\pi$ ($\cong 57.4$)

10. $O\hat{T}A = 90°$

$AT = \sqrt{12^2 - 6^2} = 6\sqrt{3}$ $\qquad T\hat{O}A = 60°$

Area = area of triangle – area of sector $= \frac{1}{2} \times 6 \times 6\sqrt{3} - \frac{60}{360}\pi \times 6^2 = 18\sqrt{3} - 6\pi = 12.3$ cm^2

OR

$T\hat{O}A = 60°$

Area of $\Delta = \frac{1}{2} \times 6 \times 12 \times \sin 60 = 18\sqrt{3}$ $\qquad$ Area of sector $= \frac{60}{360}\pi \times 6^2 = 6\pi$

Shaded area $= 18\sqrt{3} - 6\pi = 12.3$ cm^2 (3 sf)

11. Area sector OAB $=\frac{50}{360}\pi\times5^2=\frac{125}{36}\pi$ ($\cong 10.9$)

$\cos 50° = ON/5 \Rightarrow ON = 5\cos 50° \Rightarrow ON = 3.2139380\ldots$

Area of $\Delta AON = \frac{1}{2} ON \times 5\times\sin 0.8 = 6.15504\ldots$

Shaded area = $\frac{125}{36}\pi - 6.15504\ldots = 4.75$

12. (a) $r^2 + r^2 = 100 \Rightarrow r^2 = 50 \Rightarrow r = 5\sqrt{2}$

(b) circumference = $2\pi r = 10\pi\sqrt{2}$

(c) $\frac{\theta}{360}2\pi\times10 = 10\pi\sqrt{2} \Rightarrow \theta = 180\sqrt{2}$

13. (a) area of sector ABDC $=\frac{90}{360}\pi\times2^2=\pi$

area of segment BDCP = π – area of $\Delta ABC = \pi - 2$

(b) BP = $\sqrt{2}$

area of semicircle of radius BP = $\frac{1}{2}\pi(\sqrt{2})^2 = \pi$

area of shaded region = $\pi - (\pi - 2) = 2$

B. Paper 2 questions (LONG)

14. (a) $AB^2 = 12^2 + 12^2 - 2\times12\times12\times\cos 75° = 12^2(2 - 2\cos 75°)$

$= 12^2 \times 2(1 \cos 75°)$

$\Rightarrow AB = 12\sqrt{2(1-\cos 75°)}$

(b) $P\hat{O}B = 37.5°$

$BP = 12\tan 37.5° = 9.21$ cm

OR

$B\hat{P}A = 105°$ $B\hat{A}P = 37.5°$

$\frac{AB}{\sin 105°} = \frac{BP}{\sin 37.5°} \Rightarrow BP = \frac{AB\sin 37.5°}{\sin 105°} = 9.21\text{(cm)}$

(c) (i) Area $\Delta OBP = \frac{1}{2}\times12\times9.21 = 55.3\ (\text{cm}^2)$

(ii) Area $\Delta ABP = \frac{1}{2}(9.21)^2 \sin 105° = 41.0\ (\text{cm}^2)$

(d) Area of sector $=\frac{75}{360}\pi\times12^2 = 30\pi$ ($\cong 94.2$) (cm^2)

(e) Shaded area = 2 × area ΔOPB – area sector = 16.4 (cm^2)

15. (a) $l_{ACB} = \frac{310}{360} 2\pi(1) = \frac{31\pi}{18}$ ($\cong 5.41$)

(b) Area sector OACB $= \frac{310}{360} \pi \times 1^2 = \frac{31\pi}{36}$ ($\cong 2.71$)

(c) Volume = $\frac{31\pi}{36} \times 8 = \frac{62\pi}{9}$ ($\cong 21.6$)

(d) Surface area = $\frac{31\pi}{18} \times 8 + 2 \times \frac{31\pi}{36} + 2 \times 8 = \frac{31\pi}{2} + 16$ ($\cong 64.7$)

16. (a) $\frac{\text{AD}}{\sin 40°} = \frac{4}{\sin 20°} \Rightarrow$ AD = 7.51754… ≅ 7.52

(b) OAD = 180° – 60° = 120°

EITHER $\text{OD}^2 = 7.51754^2 + 4^2 - 2 \times 7.51754 \times 4 \times \cos 120° \Rightarrow$ OD = 10.12835 ≅ 10.1

OR. $\frac{\text{OD}}{\sin 120°} = \frac{4}{\sin 20°} \Rightarrow$ OD = 10.12835 ≅ 10.1

(c) area $= \frac{40}{360} \pi \times 4^2 = \frac{16\pi}{9}$ ($\cong 5.59$)

(d) area of triangle OAD: . $A = \frac{1}{2} \times 4 \times 10.12835 \times \sin 40° = 13.02075..$

area ABCD = 13.02075..$- \frac{16\pi}{9}$ = 7.44 (cm^2)