

**EXERCISES [MAI 2.5-2.6]**  
**EXPONENTS AND LOGARITHMS - I**  
**SOLUTIONS**

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**A. Paper 1 questions (SHORT)**

**EXPONENTIAL FUNCTIONS**

1. (a)

|           |           |           |
|-----------|-----------|-----------|
| $y = 2^x$ | $y = 5^x$ | $y = e^x$ |
| (c)       | (a)       | (b)       |

|              |              |              |
|--------------|--------------|--------------|
| $y = 2^{-x}$ | $y = 5^{-x}$ | $y = e^{-x}$ |
| (d)          | (f)          | (e)          |

(b)

|                       |                    |
|-----------------------|--------------------|
| $y$ – intercept       | $y = 1$            |
| Horizontal asymptote. | $y = 0$            |
| Domain                | $x \in \mathbb{R}$ |
| Range                 | $y > 0$            |

2. (a)  $y = 1$  (b) 0.25 (c)  $b = 4$   $c = 16$  (d)  $y = 0$  (f)  $x \in \mathbb{R}$   $y > 0$

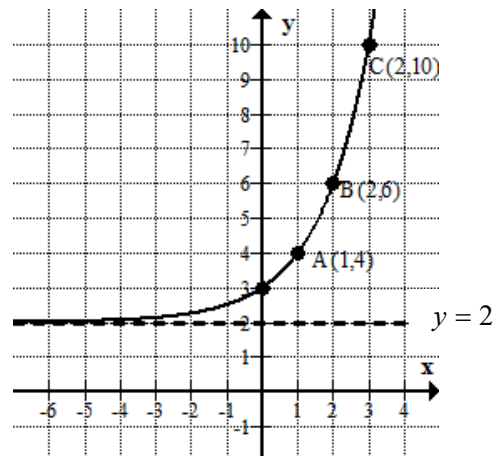
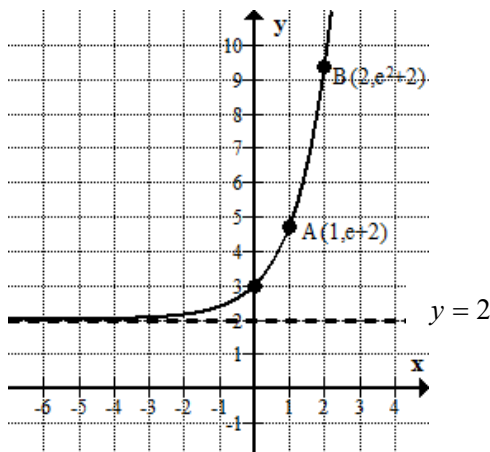
3. (a)  $b = e^2 + 2$

(b)  $y$ -intercept: (0,3), horizontal asymptote:  $y = 2$

(c) Domain:  $x \in \mathbb{R}$ , Range:  $y > 2$

4.  $y$ -intercept: (0,3), horizontal asymptote:  $y = 2$

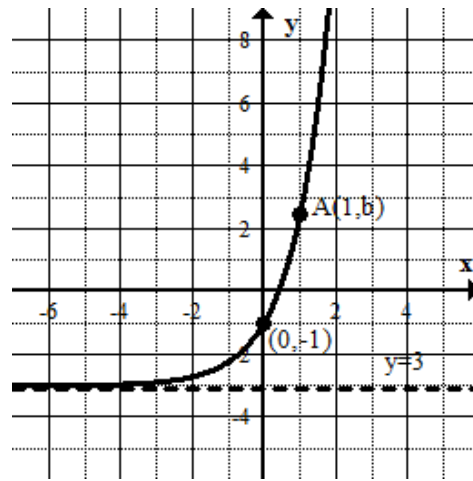
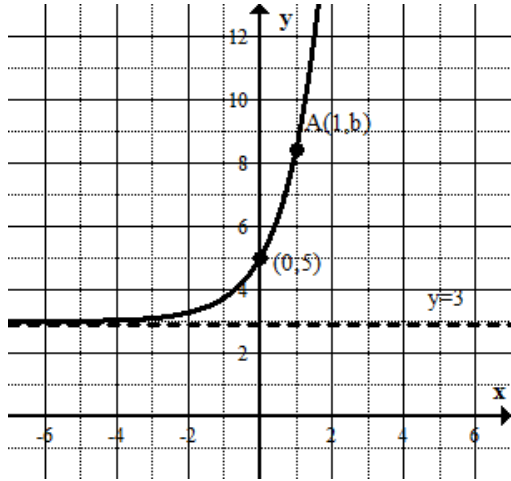
**Graphs for 3 and 4 are shown below**



5.

|                            |                       |
|----------------------------|-----------------------|
| y- intercept: (0,5)        |                       |
| h. asymptote: $y=3$        | value of $b$ : $2e+3$ |
| Domain: $x \in \mathbb{R}$ | Range: $y > 3$        |

|                            |                          |
|----------------------------|--------------------------|
| y- intercept: (0,-1)       | x- intercept: $\ln(3/2)$ |
| h. asymptote: $y= -3$      | value of $b$ : $2e-3$    |
| Domain: $x \in \mathbb{R}$ | Range: $y > -3$          |



### LOGARITHMS

6. (a)  $x = \ln 3 \cong 1.10$      $y = \log 3 \cong 0.477$     (b)  $e^x \cong 3$      $10^x \cong 3$

7.

|                           |                          |                           |
|---------------------------|--------------------------|---------------------------|
| $\log 100 = 2$            | $\log 10 = 1$            | $\log 1 = 0$              |
| $\log \frac{1}{100} = -2$ | $\log \frac{1}{10} = -1$ | $\log 0.1 = -1$           |
| $\log 10^{2020} = 2020$   | $\log \sqrt{10} = 1/2$   | $\log \sqrt[3]{10} = 1/3$ |
| $\ln 1 = 0$               | $\ln e = 1$              | $\ln e^2 = 2$             |
| $\ln \frac{1}{e} = -1$    | $\ln \frac{1}{e^2} = -2$ | $\ln \sqrt{e} = 1/2$      |

8. (a)  $x = 10^3 = 1000$     (b)  $x = e^3$     (c)  $x + 1 = 10^3 = 1000 \Leftrightarrow x = 999$

9. (a)  $k = 5$     (b)  $k = 5$     (c)  $k = 7$     (d)  $k = 7$   
 $\log xy = \log x + \log y$     and     $\ln xy = \ln x + \ln y$

10. (a)  $k = 5$     (b)  $k = 5$     (c)  $k = 7$     (d)  $k = 7$   
 $\log x^n = n \log x$     and     $\ln x^n = n \ln x$

11. (a)  $2x = 100 \Leftrightarrow x = 50$     (b)  $2x + 4 = 10 \Leftrightarrow x = 3$     (c)  $2x - 6 = 1 \Leftrightarrow x = 7/2$

12. (a)  $x = 2$   
 (b)

|  |                    |
|--|--------------------|
| $Ax = 3 \ln 2 \cong 3(0.693) \cong 2.08$ | $B = \ln 8 = 2.08$ |
|--|--------------------|

(c)

|  |                      |
|--|----------------------|
| $Ax = 3 \log 2 \cong 3(0.301) \cong 0.903$ | $B = \log 8 = 0.903$ |
|--|----------------------|

13. (a)  $y = \log_2 x$     (b) (1,0)

14. (a)  $x = -1$     (b) (i)  $\ln(0.001) = -6.91$     (ii)  $e^0 = 1$     (c) (4.64, 1.89)