EXERCISES [MAI 2.13-2.14]

POLYNOMIAL MODELS

SOLUTIONS

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A. Paper 1 questions (SHORT)

1. (a) (i) $u_n = 13 + (n-1)\times 5 = 5n+8$ (ii) $y = 5x+8$

(b) the gradient of the line is the common difference of the sequence.

(c) $5\times(-1)+8=3$, $5\times 10+8=58$, $5\times(0.2)+8=9$: All three points line on the line

(d) Although all the points lie on the line, only 58 is a term of the sequence
(it is the only one for which x is a positive integer.

2. (a) $m_{AD} = \dfrac{24}{8} = 3$

$P-2 = 3(Q-2) \Rightarrow P = 3Q-4$

(b) $P = aQ+b$. For A(2,2): $2a+b=2$

For D(10,26): $10a+b=26$

Hence: $a=3$, $b=-4$, So $P=3Q-4$

(c) For B(4,8): $3\times 4-4=8$, so it lies on the line.

For C(6,14): $3\times 6-4=14$, so it lies on the line.

3. (a) $y = 2x^2+3x+7$

(b) $y = 2(x-1)(x-5) \Rightarrow y = 2(x^2-6x+5) \Rightarrow y = 2x^2-12x+10$

(c) $y = 2(x-3)^2+5 \Rightarrow y = 2(x^2-6x+9)+5 \Rightarrow y = 2x^2-12x+23$

(d) $a(0-3)^2+2=20 \Rightarrow 9a+2=20 \Rightarrow 9a=18 \Rightarrow a=2$

Hence, $y = 2(x-3)^2+2 \Rightarrow y = 2x^2-12x+20$

4. (a) $q = -2$, $r = 4$ or $q = 4$, $r = -2$

(b) $x = 1$

(c) substituting (0, –4) into the equation: $-4 = -8p \Rightarrow p = \dfrac{4}{8}\left(=\dfrac{1}{2}\right)$

(d) $f(x) = \dfrac{1}{2}(x+2)(x-4) = \dfrac{1}{2}(x^2-2x-8) = \dfrac{1}{2}x^2 - x - 4$

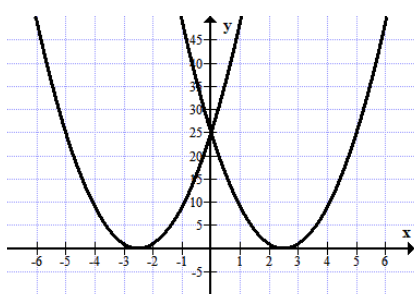
5. (a) **METHOD 1:** Discriminant = 0 $\Rightarrow q^2 - 4(4)(25) = 0 \Rightarrow q^2 = 400 \Rightarrow q = 20, q = -20$

METHOD 2: Using factorizing: $(2x-5)^2$ or $(2x+5)^2$ $\Rightarrow q = 20, q = -20$

(b) $x = 2.5$

(c) (0, 25)

(d)



6. (a) Since the vertex is at (3, 1)
$h = 3,\ k = 1$

(b) $x = 3$

(c) (5, 9) is on the graph $\Rightarrow\ 9 = a(5 - 3)^2 + 1 \Rightarrow 9 = 4a + 1 \Rightarrow a = 2$

(d) $y = 2(x - 3)^2 + 1 = 2(x^2 - 6x + 9) + 1 = 2x^2 - 12x + 19$

7. (a) $h = 3\ k = 2$

(b) $y \le 2$

(c) due to symmetry: $f(4) = a$, hence $x = 4$.

(d) $f(x) = -(x-3)^2 + 2 = -x^2 + 6x - 9 + 2 = -x^2 + 6x - 7$

8. (a) (i) $h = -1$ (ii) $k = 2$

(b) $a(1+1)^2 + 2 = 0 \Rightarrow\ a = -0.5$

(c) $y = -0.5\ (x+3)(x - 1)$

(d) $y = -0.5\ (x+3)(x - 1) = -0.5\ (x^2 + 2x - 3) = -0.5x^2 - x + 1.5$
OR $y = -0.5(x + 1)^2 + 2 = -0.5(x^2 + 2x + 1) + 2 = -0.5x^2 - x + 1.5$

9. (a) (i) $p = 1, q = 5$ (or $p = 5, q = 1$)
(ii) $x = 3$

(b) For $x = 3$, $y = -4$, hence $f(x) = (x - 3)^2 - 4$ $(h = 3, k = -4)$

(c) $f(x) = (x - 1)(x - 5) = x^2 - 6x + 5$

(d) $f(x) < 0\ \Rightarrow 1 < x < 5$

10. (a) (i) $m = 3$ (ii) $p = 2$

(b) $0 = d(1 - 3)^2 + 2 \Rightarrow\ d = -\frac{1}{2}$

(c) $f(x) = -\frac{1}{2}(x - 3)^2 + 2 = -\frac{1}{2}(x^2 - 6x + 9) + 2 = -\frac{1}{2}x^2 + 3x - \frac{5}{2}$

(d) $f(x) = -\frac{1}{2}(x - 1)(x - 5)$

11. (a) $p = -2\ \ q = 4$ (or $p = 4, q = -2$)

(b) $y = a(x+2)(x-4)$
$8 = a(6+2)(6-4)$
$8 = 16a$
$a = \frac{1}{2}$

(c) $y = \frac{1}{2}(x+2)(x-4) = \frac{1}{2}(x^2 - 2x - 8) = \frac{1}{2}x^2 - x - 4$

(d) (i) $x < -2$ or $x > 4$. (ii) $x = 5$

12. (a) $f(x) = a(x + 4)(x - 6)$
For $x = 0$, y = 240: $-24a = 240 \Rightarrow a = -10$
$f(x) = -10\ (x + 4)(x - 6)$

(b) Vertex at $x = 1$, $y = -10(1 + 4)(1 - 6) = 250$
$f(x) = -10(x - 1)^2 + 250$

(c) $y = -10(x - 1)(x - 1) + 250 = 240 + 20x - 10x^2$
OR $y = -10(x^2 - 2x + 1) + 250 = 240 + 20x - 10x^2$

13. (a) substituting (–4, 3): $3 = a(-4)^2 + b(-4) + c \Rightarrow 16a - 4b + c = 3$

(b) $36a + 6b + c = 3,\ 4a - 2b + c = -1$

(c) $a = 0.25$, $b = -0.5$, $c = -3$ (accept fractions)

$f(x) = 0.25x^2 - 0.5x - 3$

(d) Vertex (min with GDC) at (1, –3.25)

$f(x) = 0.25(x - 1)^2 - 3.25$

14. (a) $a + b + c = 4$

$4a + 2b + c = 7$

$9a + 3b + c = 14$

(b) $P = 2Q^2 - 3Q + 5$

15. (a) $4a - 2b + c = 0$

$a + b + c = 0$

$4a + 2b + c = 12$

Hence, $P = 3Q^2 + 3Q - 6$

(b) $P = a(Q + 2)(Q - 1)$

Since (2,12) lies on the line: $a(2+2)(2-1) = 12 \Rightarrow 4a = 12 \Rightarrow a = 3$

Hence, $P = 3(Q+2)(Q-1) = 3(Q^2 + Q - 2) = 3Q^2 + 3Q - 6$

16. (a) (i) Due to symmetry the vertex is V(3,10), (ii) P-intercept (0,1)

(a) $0a + 0b + c = 1$

$9a + 3b + c = 10$

$36a + 6b + c = 1$

Hence, $P = -Q^2 + 6Q + 1$

(b) $P = a(Q - 3)^2 + 10$

Since (0,1) lies on the line: $9a + 10 = 1 \Rightarrow a = -1$

Hence, $P = -(Q-3)^2 + 10 = -(Q^2 - 6Q + 9) + 10 = -Q^2 + 6Q + 1$

17. (a) $-8 = -8p + 4q - 2r$,

$-2 = p + q + r$,

$0 = 8p + 4q + 2r$

(b) $p = 1$, $q = -1$, $r = -2$

(c) $f(x) = x^3 - x^2 - 2x$,

Roots: $x = -1$, $x = 0$, $x = 2$

(d) $-1 \le x \le 0$ or $x \ge 2$.

18. (a)

x	y	**Model L$_1$**		**Model L$_2$**	
		$y_1 = 3x$	$(y - y_1)^2$	$y_2 = 3.4x - 1$	$(y - y_2)^2$
1	**3**	3	0	2.4	0.36
2	**5**	6	1	5.8	**0.64**
3	**9**	**9**	**0**	**9.2**	0.04
4	**13**	**12**	**1**	**12.6**	**0.16**
		SUM→	**2**	**SUM→**	**1.2**

(b) Although L$_1$ passes through two of the points, the model L$_2$ is closer to the points,

(c) The regression line is $y = 3.4x - 1$, that is the line L$_2$.

B. Paper 2 questions (*LONG*)

19. (a) (i) $m_{AB} = \frac{6}{2} = 3$ (ii) $y - 5 = 3(x-1) \Rightarrow y = 3x + 2$

(b) The point C(6,20) also lies on the line since

$3 \times 6 + 2 = 20$

Hence the three points lie on the same line.

(c) The point D(5,15) does not line on the line since

$3 \times 5 + 2 = 17 \neq 15$

(d) It is a satisfactory model since it passes through 3 out of the four points, and it is close enough to the fourth point.

(e) $y = 2.83x + 2.14$

(f)

x	y	Estimations		Squared Residuals	
		y_1 by line AB)	y_2 by line L	$(y - y_1)^2$	$(y - y_2)^2$
1	**5**	5	**4.97**	0	**0.0009**
3	**11**	11	**10.63**	0	**0.1369**
5	**15**	**17**	**16.29**	**4**	**0.5041**
6	**20**	**20**	**19.12**	**0**	**0.7744**
SS_{res} = SUM OF THE SQUARED RESIDUALS →				**4**	**1.4163**

(g) Although the first model passes through 3 out of the four points, the second model is much better. It is totally closer to the points since, since SS_{res} is smaller.

20. (a) (i) $P = 3.80Q + 11.6$

(ii) $P = 0.839Q^2 - 2.08Q + 17.5$

(iii) $P = -0.376Q^3 + 4.79Q^2 - 12.4Q + 21.4$

(b) exact value = 20,

Linear model value = 11.6, Percentage error = $\left|\frac{11.6 - 20}{20}\right| \times 100\% = 42\%$

Quadratic model value = 17.5, Percentage error = $\left|\frac{17.5 - 20}{20}\right| \times 100\% = 12.5\%$

Cubic model value = 21.4, Percentage error = $\left|\frac{21.4 - 20}{20}\right| \times 100\% = 7\%$

(c) The cubic model seems to be much closer to the given points.

(d) For $Q = 8$, the cubic model gives $P = 36.2$. This seems to be the most reliable prediction. (The first two models give 42 and 54.6 respectively)