

**EXERCISES [MAI 2.11-2.12]**  
**EXPONENTS AND LOGARITHMS - II**  
**SOLUTIONS**

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**A. Paper 1 questions (SHORT)**

**LOGARITHMS**

1.

$\log_2 16 = 4$	$\log_3 9 = 2$	$\log_3 27 = 3$
$\log_5 1 = 0$	$\log_5 5 = 1$	$\log_5 25 = 2$
$\log 1 = 0$	$\log 10 = 1$	$\log 10000 = 4$
$\log_3 (1/3) = -1$	$\log_3 \sqrt{3} = 1/2$	$\log 0.1 = -1$

2.

- (a) (i)  $x = 8$                       (ii)  $x = 2$   
 (b) (i)  $x = 25$                       (ii)  $x = 5$   
 (c) (i)  $2x = 16 \Leftrightarrow x = 8$       (ii)  $4x^2 = 16 \Leftrightarrow x = 2$   
 (d) (i)  $x = 100$                       (ii)  $x = e^4$

3.

$\log 10^k = k$	$\ln e^k = k$
$\log \frac{1}{10^k} = -k$	$\ln \frac{1}{e^k} = -k$
$\log \sqrt[k]{10} = 1/k$	$\ln \sqrt[k]{e} = 1/k$

4.

- (a)  $x = 8$     (b)  $x = 99$     (c)  $x = e^2 - 1$

5.

- (a) (i)  $x + 5 = 1 \Leftrightarrow x = -4$       (ii)  $x + 5 = 7 \Leftrightarrow x = 2$   
 (b) (i)  $x + 5 = 1 \Leftrightarrow x = -4$       (ii)  $x + 5 = 10 \Leftrightarrow x = 5$   
 (c) (i)  $x + 5 = 1 \Leftrightarrow x = -4$       (ii)  $x + 5 = e \Leftrightarrow x = e - 5$

6.

the answer for all questions is 5

7.

$\log xy = \log x + \log y$	LHS = $\log 100000 = 5$
	RHS = $\log 1000 + \log 100 = 3 + 2 = 5$
$\log \frac{x}{y} = \log x - \log y$	LHS = $\log 10 = 1$
	RHS = $\log 1000 - \log 100 = 3 - 2 = 1$
$\log x^2 = 2 \log x$	LHS = $\log 1000^2 = \log 1000000 = 6$
	RHS = $2 \log 1000 = 2 \cdot 3 = 6$

8.

$\ln xy = \ln x + \ln y$	LHS = $\ln 12 \cong 2.48$
	RHS = $\ln 6 + \ln 2 = 1.7918 + 0.6931 \cong 2.48$
$\ln \frac{x}{y} = \ln x - \ln y$	LHS = $\ln 3 \cong 1.10$
	RHS = $\ln 6 - \ln 2 \cong 1.7918 - 0.6931 \cong 1.7918 \cong 1.10$
$\ln_2 x^2 = 2 \ln x$	LHS = $\ln 36 \cong 3.58$
	RHS = $2 \ln 6 = 2(1.792) \cong 3.58$

9.

$\log xy$	$a+b$
$\log \frac{x}{y}$	$a-b$
$\log x^3$	$3a$
$\log xyz$	$a+b+c$
$\log x^2y$	$2a+b$
$\log \sqrt{x}$	$a/2$
$\log \frac{xy}{z}$	$a+b-c$

$\log 10x$	$a+1$
$\log 100x$	$a+2$
$\log \frac{y}{10}$	$b-1$
$\log \frac{y}{100}$	$b-2$
$\log \frac{xy}{10z}$	$a+b-c-1$
$\log \frac{1}{z}$	$-c$
$\log \frac{x^2y^7}{\sqrt{z}}$	$2a+7b-c/2$

10. The answers are exactly the same as in Exercise 9,

11.  $2 \log x + \frac{1}{2} \log y - 3 \log z = 2a + \frac{1}{2}b - 3c$

12.  $\log_{10} \left( \frac{x}{y^2 \sqrt{z}} \right) = \log_{10} x - \log_{10} y^2 - \log_{10} \sqrt{z} = \log_{10} x - 2 \log y - \frac{1}{2} \log z = p - 2q - \frac{1}{2}r$

13. (a)  $\ln a^3b = 3 \ln a + \ln b = \ln a^3b = 3p + q$

(b)  $\ln \frac{\sqrt{a}}{b} = \frac{1}{2} \ln a - \ln b = 1/2 p - q$

14. (a)  $\log_a 10 = \log_a (5 \times 2) = \log_a 5 + \log_a 2 = p + q$

(b)  $\log_a 8 = \log_a 2^3 = 3 \log_a 2 = 3q$

(c)  $\log_a 2.5 = \log_a \frac{5}{2} = \log_a 5 - \log_a 2 = p - q$

15. (a) (i)  $\log_c 15 = \log_c 3 + \log_c 5 = p + q$

(ii)  $\log_c 25 = 2 \log_c 5 = 2q$

(b)  $d^{\frac{1}{2}} = 6 \Rightarrow d = 36$

16. (a)  $\log_5 x^2 = 2 \log_5 x = 2y$

(b)  $\log_5 \frac{1}{x} = -\log_5 x = -y$

(c)  $\log_5 x^{1/2} = \frac{1}{2}y$

### EXPONENTIAL EQUATIONS

17. (a) (i)  $2x = 2 \Leftrightarrow x = 1$     (ii)  $2x = 2 \Leftrightarrow x = 1$
- (b) (i)  $2x = \log_3 5 \Leftrightarrow x = \frac{\log_3 5}{2}$
- (ii)  $2x \log 3 = \log 5 \Leftrightarrow x = \frac{\log 5}{2 \log 3}$
- (iii)  $2x \ln 3 = \ln 5 \Leftrightarrow x = \frac{\ln 5}{2 \ln 3}$
- (c)  $10^{2x} = 5 \Leftrightarrow 2x = \log 5 \Leftrightarrow x = \frac{\log 5}{2}$
- (d)  $e^{2x} = 5 \Leftrightarrow 2x = \ln 5 \Leftrightarrow x = \frac{\ln 5}{2}$
18. (a)  $x = \log_2 5 - 3$     (b)  $x = \log 5 - 3$     (c)  $x = \ln 5 - 3$     (d)  $x = \ln \frac{5}{2} - 3$

### LOGARITHMIC EQUATIONS

19. (a)  $\log x(x+1) = \log 6 \Leftrightarrow x(x+1) = 6 \Leftrightarrow x^2 + x - 6 = 0 \Leftrightarrow x = 2$  ( $-3$  is rejected)
- (b)  $\log x(x+3) = 1 \Leftrightarrow x(x+3) = 10 \Leftrightarrow x^2 + 3x - 10 = 0 \Leftrightarrow x = 2$  ( $-5$  is rejected)
- (c)  $\log \frac{x+18}{x} = 1 \Leftrightarrow \frac{x+18}{x} = 10 \Leftrightarrow x+18 = 10x \Leftrightarrow 18 = 9x \Leftrightarrow x = 2$
20. (a)  $\log_2 x(x+1) = \log_2 6 \Leftrightarrow x(x+1) = 6 \Leftrightarrow x^2 + x - 6 = 0 \Leftrightarrow x = 2$  ( $-3$  is rejected)
- (b)  $\log_2 x(x+1) = 1 \Leftrightarrow x(x+1) = 2 \Leftrightarrow x^2 + x - 2 = 0 \Leftrightarrow x = 1$  ( $-2$  is rejected)
- (c)  $\log_2 \frac{x+5}{x} = 1 \Leftrightarrow \frac{x+5}{x} = 2 \Leftrightarrow x+5 = 2x \Leftrightarrow x = 5$
21. (a)  $\log \frac{10x+20}{x^2} = 1 \Leftrightarrow \frac{10x+20}{x^2} = 10 \Leftrightarrow 10x+20 = 10x^2 \Leftrightarrow x+2 = x^2 \Leftrightarrow x^2 - x - 2 = 0 \Leftrightarrow x = 2$
- (a)  $\log_2 \frac{x+14}{x^2} = 2 \Leftrightarrow \frac{x+14}{x^2} = 4 \Leftrightarrow x+14 = 4x^2 \Leftrightarrow 4x^2 - x - 14 = 0 \Leftrightarrow x = 2$  ( $-7/4$  is rejected)
- (b)  $\log_2 (4x)(x^2) = 5 \Leftrightarrow 4x^3 = 32 \Leftrightarrow x^3 = 8 \Leftrightarrow x = 2$
22.  $\log_2 x(x-2) = 3$   
 $x(x-2) = 2^3, x^2 - 2x - 8$   
 $x = \frac{2 \pm \sqrt{36}}{2}$   
 $x = 4$
23. (a)  $3x + 5 = a^2 \quad x = \frac{a^2 - 5}{3}$
- (b)  $x + 2 = e^3 \Leftrightarrow x = e^3 - 2$  ( $= 18.1$ )

**B. Paper 2 questions (LONG)**

24. (a)  $x^2 = 49 \quad x = \pm 7 \quad x = 7$

(b)  $2^x = 8 \quad x = 3$

(c)  $x = 25^{-\frac{1}{2}} \quad x = \frac{1}{\sqrt{25}} \quad x = \frac{1}{5}$

(d)  $\log_2(x(x-7)) = 3$

$\log_2(x^2 - 7x) = 3$

$2^3 = x^2 - 7x$

$x^2 - 7x - 8 = 0$

$x = 8, x = -1$  (rejected)

$x = 8$

25. (a)  $x \in R$

(b)  $y = 15$

(c)  $f(5) \cong 49.8$

(d)  $x \cong 7.50$

(e)  $x > 8.14$  so  $x = 9$

(f)  $y \cong 5.02$ , the horizontal line is  $y = 5$

(g)  $y > 5$

26. (a)  $f^{-1}(x) = \ln x$

(b)  $g^{-1}(x) = \frac{e^x - 1}{2}$

(c)  $(f \circ g)(x) = e^{\ln(1+2x)} = 1 + 2x$

(d)  $y = 2x + 1 \Leftrightarrow y - 1 = 2x \Leftrightarrow x = (y-1)/2$  so  $(f \circ g)^{-1}(x) = \frac{x-1}{2}$

27. (a)  $a = 135$

(b) 1.61 2.71 3.81 4.91

(c) all differences are 1.1 they are in arithmetic sequence

(d) 0.699 1.18 1.65 2.13

The differences are 0.481 0.47 0.48

The values are almost equal; the difference is due to rounding. In fact we still have an arithmetic sequence.

(e)  $a \quad 3a \quad 9a$

(f)  $\ln 3a - \ln a = \ln \frac{3a}{a} = \ln 3 \quad \ln 9a - \ln 3a = \ln \frac{9a}{3a} = \ln 3 \quad \text{common difference} = \ln 3$

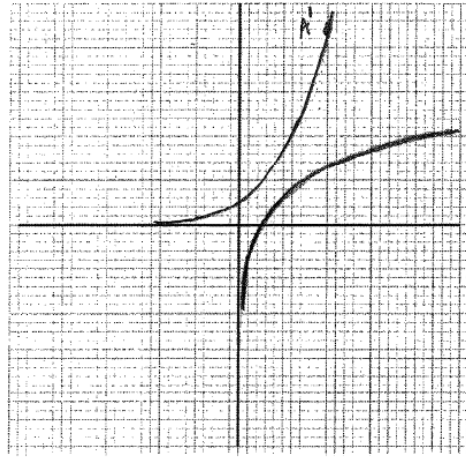
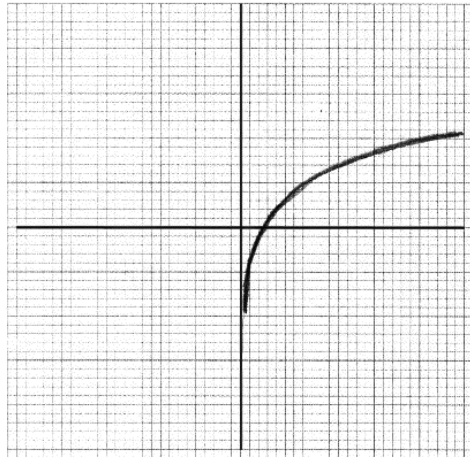
(g) If the sequence  $u_n$  is geometric then the sequence  $v_n = \ln u_n$  is arithmetic

$u_n = u_1 r^{n-1} \quad \text{and} \quad u_{n+1} = u_1 r^n$

$v_{n+1} - v_n = \ln v_{n+1} - \ln v_n = \ln u_1 r^n - \ln u_1 r^{n-1} = \ln \frac{u_1 r^n}{u_1 r^{n-1}} = \ln r$

So the common difference is  $d = \ln r$

28. (a)  $f(x) = \log_3 \frac{x}{2} = \log_3 \frac{8x}{4}$   $f(x) = \log_3 2x$   
 (b)  $f(0.5) = 0, f(4.5) = 2$   
 (c) and (e)



V.A.  $x = 0$  (must be an equation)

- (d)  $f^{-1}(0) = 0.5$