EXERCISES [MAI 1.14]

MATRIX EQUATIONS – THE LINEAR SYSTEM *AX*=*B*

SOLUTIONS

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A. Paper 1 questions (SHORT)

MATRIX EQUATIONS

1. (i) $2\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}+3\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}\Leftrightarrow\begin{pmatrix}2+3a & 4+3b\\ 6+3c & 8+3d\end{pmatrix}=\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}$

$2+3a=2 \Leftrightarrow a=0$,

$4+3b=2 \Leftrightarrow b=-\dfrac{2}{3}$,

$6+3c=2 \Leftrightarrow c=-\dfrac{4}{3}$,

$8+3d=2 \Leftrightarrow d=-2$.

Thus $X=\begin{pmatrix}0 & -2/3\\ -4/4 & -2\end{pmatrix}$

(ii) $X=\dfrac{1}{3}B-\dfrac{2}{3}A=\begin{pmatrix}0 & -2/3\\ -4/4 & -2\end{pmatrix}$

2. (a) $A^{-1}=\dfrac{1}{-2}\begin{pmatrix}4 & -2\\ -3 & 1\end{pmatrix}=\begin{pmatrix}-2 & 1\\ 3/2 & -1/2\end{pmatrix}$

(b) _(i) $AX=B\Leftrightarrow\begin{pmatrix}a+2c & b+2d\\ 3a+4c & 3b+4d\end{pmatrix}=\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}$

$\Leftrightarrow$ $\begin{matrix}a+2c=2\\ 3a+4c=2\end{matrix}$ and $\begin{matrix}b+2d=2\\ 3b+4d=2\end{matrix}$

Then, $a=-2$, $b=-2$, $c=2$, $d=2$.

Thus $X=\begin{pmatrix}-2 & -2\\ 2 & 2\end{pmatrix}$

(ii) $X=A^{-1}B=\begin{pmatrix}-2 & 1\\ 3/2 & -1/2\end{pmatrix}\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}=\begin{pmatrix}-2 & -2\\ 2 & 2\end{pmatrix}$

(c) $X=BA^{-1}=\begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}\begin{pmatrix}-2 & 1\\ 3/2 & -1/2\end{pmatrix}=\begin{pmatrix}-1 & 1\\ -1 & 1\end{pmatrix}$

3.

Equation	Solve for X
$X - A = B$	$X = A + B$
$A + 2X = B$	$X = (1/2)(B - A)$
$AX = C$	$X = A^{-1}C$
$XA = C$	$X = C A^{-1}$
$AXA = C$	$X = A^{-1}C A^{-1}$
$AX - B = C$	$X = A^{-1}(B + C)$
$2AX + 3B = C$	$X = (1/2)A^{-1}(C - 3B)$

4. (a) $A^{-1}XB = C \Rightarrow X = ACB^{-1}$

(analytically:

$$A^{-1}XB = C \Rightarrow AA^{-1}XB = AC \Rightarrow IXB = AC \Rightarrow XB = AC$$

$$\Rightarrow XBB^{-1} = ACB^{-1} \Rightarrow XI = ACB^{-1} \Rightarrow X = ACB^{-1} \quad)$$

(b) (i) $AX - X = C \Leftrightarrow (A-I)X = C \Leftrightarrow X = (A-I)^{-1}C$

(ii) $XA - X = C \Leftrightarrow X(A-I) = C \Leftrightarrow X = C(A-I)^{-1}$

5. (a) $A^2 = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -8 \\ 0 & 9 \end{pmatrix}.$

(b) $3X + A = B \Leftrightarrow 3X = B - A \Leftrightarrow X = \frac{1}{3}(B - A) \Leftrightarrow X = \frac{1}{3}\begin{pmatrix} -4 & 6 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} -4/3 & 2 \\ 2/3 & -2/3 \end{pmatrix}$

6. $X = B^{-1}(A - AB)$

EITHER $A - AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 4 & -9 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} -2 & 12 \\ -1 & -8 \end{pmatrix}$

$$X = B^{-1}(A - AB) = B^{-1}\begin{pmatrix} -2 & 12 \\ -1 & -8 \end{pmatrix} = -\frac{1}{6}\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -2 & 12 \\ -1 & -8 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ \frac{1}{3} & \frac{8}{3} \end{pmatrix}$$

OR directly by GDC $X = B^{-1}(A - AB) = \begin{pmatrix} -1 & 6 \\ \frac{1}{3} & \frac{8}{3} \end{pmatrix}$

7. $B = (BA)A^{-1} = -\frac{1}{4}\begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -2 & 5 \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} -4 & -12 \\ -16 & -48 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}$

OR

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$

$\left.\begin{array}{l} 5a + 2b = 11 \\ 2a \quad\quad = 2 \end{array}\right\} \Rightarrow a = 1, b = 3$

$\left.\begin{array}{l} 5c + 2d = 44 \\ 2c \quad\quad = 8 \end{array}\right\} \Rightarrow c = 4, d = 12$

$B = \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}$

8. (a) $\boldsymbol{AB} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$

(b) $\boldsymbol{A}^{-1}\boldsymbol{X} = \boldsymbol{B} \Rightarrow \boldsymbol{AA}^{-1}\boldsymbol{X} = \boldsymbol{AB} \Rightarrow \boldsymbol{X} = \boldsymbol{AB} \Rightarrow \boldsymbol{X} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$

9. (a) (i) $\boldsymbol{AB} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ $(= 4\boldsymbol{I})$

(ii) $\boldsymbol{A}^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}$ **OR** $\frac{1}{4}\boldsymbol{B}$ **OR** $\begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{5}{4} \end{pmatrix}$

(b) **METHOD 1**

$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{A}^{-1}\boldsymbol{C} = \begin{pmatrix} 5 \\ -17 \end{pmatrix}$

METHOD 2

By using $\boldsymbol{AX} = \boldsymbol{C}$: $5x + y = 8$, $6x + 2y = -4$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -17 \end{pmatrix}$

10. (a) $\det \boldsymbol{A} = 5(1) - 7(-2) = 19$, $\quad \boldsymbol{A}^{-1} = \frac{1}{19}\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$

OR $\boldsymbol{A}^{-1} = \begin{pmatrix} 0.0526 & 0.105 \\ -0.368 & 0.263 \end{pmatrix}$

(b) (i) $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C} \Rightarrow \boldsymbol{XA} = \boldsymbol{C} - \boldsymbol{B} \Rightarrow \boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1}$

(ii) $\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1} = \boldsymbol{X} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$

11. (a) det $\boldsymbol{A} = 2$

$$\boldsymbol{A}^{-1} = \frac{1}{2}\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \quad \left(= \begin{pmatrix} \frac{3}{2} & 2 \\ \frac{1}{2} & 1 \end{pmatrix} \right)$$

(b) $\boldsymbol{X} = \boldsymbol{A}^{-1}\begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix} \Rightarrow \boldsymbol{X} = \begin{pmatrix} 10 & 5 \\ 4 & 1 \end{pmatrix}$

12. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{7}{3} & \frac{5}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -7 & 5 \end{pmatrix}$ or $\begin{pmatrix} 0.667 & -0.333 \\ -2.33 & -1.67 \end{pmatrix}$

(b) $\boldsymbol{AX} = \boldsymbol{C} - \boldsymbol{B}$

$\boldsymbol{X} = \boldsymbol{A}^{-1}(\boldsymbol{C} - \boldsymbol{B})$

$\boldsymbol{D} = \boldsymbol{C} - \boldsymbol{B} = \begin{pmatrix} 7 & -11 \\ 11 & -13 \end{pmatrix}$

(a) $\boldsymbol{X} = \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}$

13. $\begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}\boldsymbol{X} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\boldsymbol{X} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$. Thus $\begin{pmatrix} 4 & 1 \\ -5 & 7 \end{pmatrix}\boldsymbol{X} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$

$$\boldsymbol{X} = \begin{pmatrix} 4 & 1 \\ -5 & 7 \end{pmatrix}\begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix} = \frac{1}{33}\begin{pmatrix} 28 & 59 \\ 20 & 28 \end{pmatrix}$$

OR $\boldsymbol{AX} + \boldsymbol{X} = \boldsymbol{B} \Rightarrow (\boldsymbol{A} + \boldsymbol{I})\boldsymbol{X} = \boldsymbol{B} \Rightarrow \boldsymbol{X} = (\boldsymbol{A} + \boldsymbol{I})^{-1}\boldsymbol{B} = \frac{1}{33}\begin{pmatrix} 28 & 59 \\ 20 & 28 \end{pmatrix}$

$$a = \frac{28}{33}, b = \frac{59}{33}, c = \frac{20}{33}, d = \frac{28}{33}$$

14. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} 3 & 2 & -3 \\ 2 & 1 & -2 \\ -8 & -6 & 9 \end{pmatrix}$

(b) $\boldsymbol{AB} = \begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix} - \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix}, = \begin{pmatrix} 10 & 4 & -8 \\ 1 & 2 & -12 \\ 10 & 5 & -15 \end{pmatrix}$

$$\boldsymbol{B} = \boldsymbol{A}^{-1}\begin{pmatrix} 10 & 4 & -8 \\ 1 & 2 & -12 \\ 10 & 5 & -15 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 0 & 2 \\ 4 & 1 & 1 \end{pmatrix}$$

THE LINEAR SYSTEM *AX*=*B*

15. (a) (i) det $\boldsymbol{A}$ = 21-16=5

(ii) $\boldsymbol{A}^{-1} = \frac{1}{5}\begin{pmatrix} 3 & -8 \\ -2 & 7 \end{pmatrix}$

(b) Set up matrix equation $\begin{pmatrix} 7 & 8 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{A}^{-1}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 3 & -8 \\ -2 & 7 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -5 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$x = -1, y = 1$

16. (a) det $\boldsymbol{M} = -4$

(b) $\boldsymbol{M}^{-1} = -\frac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix} \left(= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right)$

(c) $\boldsymbol{X} = \boldsymbol{M}^{-1}\begin{pmatrix} 4 \\ 8 \end{pmatrix} \quad \left(\boldsymbol{X} = -\frac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 8 \end{pmatrix} \right)$

$\boldsymbol{X} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(d) 2x + y = 4

2x − y = 8.

$(x=3, y=-2)$

17. (a) **METHOD 1**

$\boldsymbol{M} = (\boldsymbol{M}^{-1})^{-1}$

$\boldsymbol{M} = \frac{1}{10}\begin{pmatrix} 2 & 0 \\ -1 & 5 \end{pmatrix}$

METHOD 2

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$5a + b = 1$, $2b = 0$, $5c + d = 0$, $2d = 1$

$\boldsymbol{M} = \begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix}$

(b) **METHOD 1**

$\boldsymbol{X} = \boldsymbol{M}^{-1}\boldsymbol{B}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \end{pmatrix}$

METHOD 2

$\begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \quad \Rightarrow \quad 0.2x = 1, \quad -0.1x + 0.5y = 7$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \end{pmatrix}$

18. (a) $A^{-1} = \begin{pmatrix} -4.33 & -2 & 1.67 \\ 1.67 & 1 & -0.333 \\ -0.667 & 0 & 0.333 \end{pmatrix} \quad \left(= \begin{pmatrix} -\frac{13}{3} & -2 & \frac{5}{3} \\ \frac{5}{3} & 1 & -\frac{1}{3} \\ -\frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} \right)$

(b) $X = A^{-1}B = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$ (accept $x = 1, y = 0, z = -1$)

19. (a) $A^{-1} = \begin{pmatrix} 0.1 & 0.4 & 0.1 \\ -0.7 & 0.2 & 0.3 \\ -1.2 & 0.2 & 0.8 \end{pmatrix}$

(b) the equations may be written as $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \left(= \begin{pmatrix} 1.2 \\ 0.6 \\ 1.6 \end{pmatrix} \right)$

$x = 1.2, y = 0.6, z = 1.6$

20. (a) $A^{-1} = \begin{pmatrix} -0.2 & 1.8 & -0.6 \\ -0.4 & 0.6 & -0.2 \\ 0.4 & -2.6 & 1.2 \end{pmatrix}$

(b) the equations may be written as $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathrm{A}^{-1}\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$x = 4, y = 1, z = -6$

21. (a) $A^{-1} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{5}{3} & -\frac{7}{3} \\ \frac{2}{3} & -\frac{4}{3} & \frac{5}{3} \end{pmatrix}$ or $\begin{pmatrix} -0.333 & 0.667 & -0.333 \\ -0.333 & 1.67 & -2.33 \\ 0.667 & -1.33 & 1.67 \end{pmatrix}$

(b) (i) $X = A^{-1}B$

(ii) $X = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$

22. (a) $\boldsymbol{M} = \begin{pmatrix} 1 & 6 & -3 \\ 4 & 2 & -4 \\ 1 & 1 & 5 \end{pmatrix}, \boldsymbol{N} = \begin{pmatrix} -1 \\ 12 \\ 15 \end{pmatrix}$

(b) $\boldsymbol{X} = \boldsymbol{M}^{-1}\boldsymbol{N}$

$\boldsymbol{X} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$

(c) $x = 5, y = 0, z = 2$

23. (a) (i) $a = 4$

(ii) $b = 7$

(b) $\boldsymbol{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ -12 \end{pmatrix}$

(c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{A}^{-1}\begin{pmatrix} 7 \\ 10 \\ -12 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix}$ ($x = -3, y = 5, z = 4$)

B. Paper 2 questions (LONG)

24. (a) (i) $\boldsymbol{A}^{-1} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$

(ii) $\boldsymbol{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix} + \begin{pmatrix} p & 2 \\ 0 & q \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 4 & 3 \end{pmatrix}$

$p = 2, q = 3$

(c) $\boldsymbol{A}^{-1}\boldsymbol{B} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & \frac{3}{2} \\ 1 & 1 \end{pmatrix}$

(d) $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$

$\boldsymbol{X} = \begin{pmatrix} 0 & \frac{3}{2} \\ 1 & 1 \end{pmatrix}$

25. (a) $M^2 = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} a^2+4 & 2a-2 \\ 2a-2 & 5 \end{pmatrix}$

(b) $2a - 2 = -4 \Rightarrow a = -1$

(then also $a^2 + 4 = 5$)

Note: *You may solve $a^2 + 4 = 5$ to give $a = \pm 1$,*

and then show that only $a = -1$ satisfies $2a - 2 = -4$.

(c) $M = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$

$M^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{pmatrix}$

The system may be written

$\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{pmatrix}\begin{pmatrix} -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$x = 1 \quad y = -1$

26. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} -1 & 1 & -1 \\ 1 & -0.5 & 1.25 \\ 1 & -0.5 & 0.75 \end{pmatrix}$

(b) (i) $\boldsymbol{I} - \frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1} \Rightarrow -\frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1} - \boldsymbol{I} \Rightarrow \boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$

(ii) $\boldsymbol{B} = \begin{pmatrix} 4 & -2 & 2 \\ -2 & 3 & -2.5 \\ -2 & 1 & 0.5 \end{pmatrix}$

(iii) det $\boldsymbol{B} = 12$

(iv) det $\boldsymbol{B} \neq 0$

(c) (i) $\boldsymbol{X} = \boldsymbol{B}^{-1}\boldsymbol{C} = \begin{pmatrix} 0.333 \\ 1 \\ 1.33 \end{pmatrix} \left(= \begin{pmatrix} \frac{1}{3} \\ 1 \\ \frac{4}{3} \end{pmatrix} \right)$

(ii) $4x - 2y + 2z = 2$

$-2x + 3y - 2.5z = -1$

$-2x + y + 0.5z = 1$

27. (a) substituting (–4, 3)

$3 = a(-4)^2 + b(-4) + c$

$16a - 4b + c = 3$

(b) $36a + 6b + c = 3$

$4a - 2b + c = -1$

(c) (i) $\boldsymbol{A} = \begin{pmatrix} 16 & -4 & 1 \\ 36 & 6 & 1 \\ 4 & -2 & 1 \end{pmatrix}; \boldsymbol{B} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$

(ii) $\boldsymbol{A}^{-1} = \begin{pmatrix} 0.05 & 0.0125 & -0.0625 \\ -0.2 & 0.075 & 0.125 \\ -0.6 & 0.1 & 1.5 \end{pmatrix} \quad \left(= \begin{pmatrix} \frac{1}{20} & \frac{1}{80} & -\frac{1}{16} \\ -\frac{1}{5} & \frac{3}{40} & \frac{1}{8} \\ -\frac{3}{5} & \frac{1}{10} & \frac{3}{2} \end{pmatrix} \right)$

(iii) $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B} = \begin{pmatrix} 0.25 \\ -0.5 \\ -3 \end{pmatrix}$

$f(x) = 0.25x^2 - 0.5x - 3$

(d) $f(x) = 0.25(x - 1)^2 - 3.25$

28. (a) $q = 0$

(b) substitute (3, 18)

$m3^3 + n3^2 + p3 = 18 \Rightarrow 27m + 9n + 3p = 18$

(c) $m + n + p = 0$

$-m + n - p = -10$

(d) (i) $\begin{pmatrix} 27 & 9 & 3 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 18 \\ 0 \\ -10 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

(e) $f(x) = x(2x^2 - 5x + 3) = x(x - 1)(2x - 3)$

$r = 2 \qquad s = 3$