# matrix transformation hl p1 *[34 marks]*

The transformation $T$ is represented by the matrix $M = \begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}$.

A pentagon with an area of $12\,\text{cm}^2$ is transformed by $T$.

1a. Find the area of the image of the pentagon. *[2 marks]*

## Markscheme

attempt to find $\det(M)$ ***(M1)***

$= 14$

$(12 \times 14) = 168\ \text{cm}^2$ ***A1***

***[2 marks]***

Under the transformation $T$, the image of point $\text{X}$ has coordinates $(2t - 3,\ 6 - 5t)$, where $t \in \mathbb{R}$.

1b. Find, in terms of $t$, the coordinates of $\text{X}$. *[6 marks]*

# Markscheme

let $\mathrm{X}$ have coordinates $(x,\ y)$

**METHOD 1**

$M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$ ***(A1)***

$M^{-1} = \frac{1}{14}\begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix}$ ***A1***

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 2t-3+24-20t \\ -6t+9+12-10t \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 21-18t \\ 21-16t \end{pmatrix}$ **OR** $\left(\frac{21-18t}{14},\ \frac{21-16t}{14}\right)$ ***A1A1***

**METHOD 2**

writing two simultaneous equations ***(M1)***

$2x-4y=2t-3$ ***(A1)***

$3x+y=6-5t$ ***(A1)***

attempting to solve the equations ***(M1)***

$(x,\ y) = \left(\frac{3}{2}-\frac{9t}{7},\ \frac{3}{2}-\frac{8t}{7}\right)$ ***A1A1***

***[6 marks]***

The equation of the line $y=mx+c$ can be expressed in vector form $r=a+\lambda b$.

2a. Find the vectors $a$ and $b$ in terms of $m$ and/or $c$. *[2 marks]*

# Markscheme

(one vector to the line is $\begin{pmatrix} 0 \\ c \end{pmatrix}$ therefore) $a = \begin{pmatrix} 0 \\ c \end{pmatrix}$ ***A1***

the line goes $m$ up for every $1$ across

(so the direction vector is) $b = \begin{pmatrix} 1 \\ m \end{pmatrix}$ ***A1***

**Note:** Although these are the most likely answers, many others are possible.

***[2 marks]***

The matrix $M$ is defined by $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$.

2b. Find the value of $\det M$. *[1 mark]*

# Markscheme

(from GDC **OR** $6 \times 2 - 4 \times 3$) $|M| = 0$ ***A1***

***[1 mark]***

The line $y = mx + c$ (where $m \neq -2$) is transformed into a new line using the transformation described by matrix $M$.

2c. Show that the equation of the resulting line does not depend on $m$ or $c$. *[4 marks]*

# Markscheme

**METHOD 1**

$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} 6x + 3mx + 3c \\ 4x + 2mx + 2c \end{pmatrix}$ ***M1A1***

$$=\begin{pmatrix} 3(2x+mx+c) \\ 2(2x+mx+c) \end{pmatrix}$$ ***A1***

therefore the new line has equation $3Y = 2X$ ***A1***

which is independent of $m$ or $c$ ***AG***

**Note:** The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

**METHOD 2**

take two points on the line, e.g $(0,\ c)$ and $(1,\ m+c)$ ***M1***

these map to $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ c \end{pmatrix}=\begin{pmatrix} 3c \\ 2c \end{pmatrix}$

and $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ m+c \end{pmatrix}=\begin{pmatrix} 6+3m+3c \\ 4+2m+2c \end{pmatrix}$ ***A1***

therefore a direction vector is $\begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix}=(2+m)\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(since $m \neq -2$) a direction vector is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

the line passes through $\begin{pmatrix} 3c \\ 2c \end{pmatrix}-c\begin{pmatrix} 3 \\ 2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ therefore it always has the origin as a jump-on vector ***A1***

the vector equation is therefore $r = \mu\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ***A1***

which is independent of $m$ or $c$ ***AG***

**Note:** The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

**METHOD 3**

$$r=\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\left(\begin{pmatrix} 0 \\ c \end{pmatrix}+\lambda\begin{pmatrix} 1 \\ m \end{pmatrix}\right)=\begin{pmatrix} 3c \\ 2c \end{pmatrix}+\lambda\begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix}$$ ***M1A1***

$$= c\begin{pmatrix} 3 \\ 2 \end{pmatrix}+(2+m)\lambda\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$ ***A1***

$$= \mu\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

where $\mu = c + (2+m)\lambda$ is an arbitrary parameter. ***A1***

which is independent of $m$ or $c$ (as $\mu$ can take any value) ***AG***

**Note:** The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

***[4 marks]***

A geometric transformation $T:\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x' \\ y' \end{pmatrix}$ is defined by

$T:\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

3a. Find the coordinates of the image of the point $(6, -2)$. *[2 marks]*

# Markscheme

$\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 6 \\ -2 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 57 \\ 22 \end{pmatrix}$ **OR** $(57, 22)$ ***A1***

***[2 marks]***

3b. Given that $T:\begin{pmatrix} p \\ q \end{pmatrix} \mapsto 2\begin{pmatrix} p \\ q \end{pmatrix}$, find the value of $p$ and the value of $q$. *[3 marks]*

# Markscheme

$\begin{pmatrix} 2p \\ 2q \end{pmatrix} = \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$7p - 10q - 5 = 2p$

$2p - 3q + 4 = 2q$ ***(A1)***

solve simultaneously:

$p = 13, q = 6$ ***A1***

**Note:** Award ***A0*** if $13$ and $6$ are not labelled or are labelled the other way around.

***[3 marks]***

3c. A triangle $L$ with vertices lying on the $xy$ plane is transformed by $T$. *[2 marks]*

Explain why both $L$ and its image will have exactly the same area.

# Markscheme

$\det \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} = -1 \left( \text{OR} \left| \det \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \right| = 1 \right)$ ***A1***

scale factor of image area is therefore $(|-1| =)1$ (and the translation does not affect the area) ***A1***

***[2 marks]***

Points in the plane are subjected to a transformation $T$ in which the point $(x, y)$ is transformed to the point $(x', y')$ where

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

4a. Describe, in words, the effect of the transformation $T$. *[1 mark]*

## Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

a stretch of scale factor $3$ in the $x$ direction

and a stretch of scale factor $2$ in the $y$ direction ***A1***

***[1 mark]***

4b. Show that the points $\mathrm{A}(1,\ 4),\ \mathrm{B}(4,\ 8),\ \mathrm{C}(8,\ 5),\ \mathrm{D}(5,\ 1)$ form a square. *[3 marks]*

## Markscheme

the four sides are equal in length $(5)$ ***A1***

$\mathrm{Grad\ AB} = \frac{4}{3},\ \mathrm{Grad\ BC} = -\frac{3}{4}$ ***A1***

so product of gradients $= -1$, therefore $\mathrm{AB}$ is perpendicular to $\mathrm{BC}$ ***A1***

therefore $\mathrm{ABCD}$ is a square ***AG***

***[3 marks]***

4c. Determine the area of this square. *[1 mark]*

## Markscheme

area of square $= 25$ ***A1***

***[1 mark]***

4d. Find the coordinates of $\mathrm{A'}$, $\mathrm{B'}$, $\mathrm{C'}$, $\mathrm{D'}$, the points to which $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ are transformed under $T$. *[2 marks]*

## Markscheme

the transformed points are

$\text{A'} = (3,\ 8)$

$\text{B'} = (12, 16)$

$\text{C'} = (24, 10)$

$\text{D'} = (15,\ 2)$ ***A2***

**Note:** Award ***A1*** if one point is incorrect.

***[2 marks]***

4e. Show that $\text{A' B' C' D'}$ is a parallelogram. *[3 marks]*

## Markscheme

$\text{Grad A'B'} = \frac{8}{9};\ \text{Grad C'D'} = \frac{8}{9}$ ***A1***

therefore $\text{A'B'}$ is parallel to $\text{C'D'}$ ***R1***

$\text{Grad A'D'} = -\frac{6}{12};\ \text{Grad B'C'} = -\frac{6}{12}$ ***A1***

therefore $\text{A'D'}$ is parallel to $\text{B'C'}$

therefore $\text{A'B'C'D'}$ is a parallelogram ***AG***

***[3 marks]***

4f. Determine the area of this parallelogram. *[2 marks]*

## Markscheme

$\text{area of parallelogram} = |\text{determinant}| \times \text{area of square}$

$= 6 \times 25$ ***(M1)***

$= 150$ ***A1***

***[2 marks]***